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THE SCIENTIFIC METHOD

The large number of requests for further illustrations of the scientific method as given in the article, "Is This the Scientific Method" by Ira C. Davis in the January number of this magazine indicates that many teachers are interested in developing this objective to a fuller extent than they have been doing. It seems desirable to run a series of articles to meet these requests. Mr. Homer W. LeSourd begins the series in this number with an article entitled, "Testing Vacuum Bottles." (See page 234.)

It is hoped many teachers will describe some specific classroom procedures, some pupil activities, or some study plans which they have used and which are designed to give pupils a better understanding of the scientific method. It is also hoped that teachers will comment on these illustrations as they appear, indicating what they consider to be the correct procedures, and if incorrect, point out the errors.

Training in the use of the scientific method has been a sort of catch phrase for science teachers for several years. It is listed as one of the major objectives in all of the recent investigations. Teachers are urged to adopt methods of teaching which will give pupils more opportunity to understand the scientific method. Can this training be given to pupils without the use of the laboratory? Can pupils learn what the scientific method is by studying the methods our scientists have used? Can they solve scientific problems if they have had no actual experience in solving them?

There is considerable misunderstanding as to the true meaning of the scientific method. No satisfactory definition is available. The procedures to be used in teaching pupils how to use

the method are vague and not fully understood. The proposed series should help in bringing to the attention of teachers the possibilities in its use, actual illustrations of its use in the classroom, and suggestions for its improvement and refinement.

TEACHING SCIENTIFIC METHOD
Article I: Testing Vacuum Bottles

By Homer W. LeSourd
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It is not difficult to obtain vacuum bottles of different makes and sizes so that each pupil in a general science class may test one. A chemical thermometer is the only additional apparatus needed. Tests may be made at home or in the laboratory. Boiling water is first placed in a bottle, its temperature is taken and the bottle is corked for 24 hours when the temperature is again taken. In like manner the bottle is again tested by using ice-cold water. The data for all these bottles is tabulated in class, each pupil recording in the form shown here, the data supplied by all pupils in the class.

Make of Bottle	Approximate Age	Size of Bottle	Test with hot water			Test with cold water		
			1st	2nd	change	1st	2nd	change

The class is then asked to propose questions which might be answered by a study of this table. Usually the following are suggested.

(a) Which particular make of bottle seems to be best and which poorest?

(b) Does a large bottle give better results than a small one?

(c) Does a bottle give better results with hot or with cold water?

(d) Does a bottle deteriorate with age?

Pupils are expected to independently write conclusions without guidance from the teacher. They are advised to write all

statements with caution. Then comes an oral discussion of the validity of these conclusions. Is it fair to judge bottles on such evidence as we have before us? Given ample funds for the purpose what bottles should be provided to make the investigation thoroughly satisfactory? Finally, to what extent can our conclusions be trusted in the purchase of vacuum bottles? Perhaps the defects in this exercise constitute a strong recommendation for its use since it is our aim to cultivate a critical attitude of mind.

WHAT GENERAL SCIENCE CAN DO FOR THE HIGH SCHOOL STUDENT*

By Joseph F. Gonnelly
Principal Hyde Park High School, Chicago

My experience as teacher and administrator has convinced me that much of the lack of effectiveness of our schools is due to vagueness in the formulation of the aims and outcomes of school activities. It is, therefore, a source of pleasure to deal with the topic assigned me for this paper. The aims must be clear—definite—compelling to both student and teacher. It is not enough that the teacher know the purposes of his subject. His students, also, must be vitally aware of these purposes. Too often our school work is relatively futile because there has been no provision for adequate motivation and for effective transfer of training.

General science, in my judgment, can initiate certain desirable changes in the high school student's nature. In what I shall say hereafter, I shall not mean that general science alone can bring about the result mentioned. But it can start that student along the right path. Certainly our work will be productive of the optimum results if we agree on what results we want—if our pupils know these results—if we can inflame their imaginations with the desirability of attaining these results.

General science should promote better health in our high school students—today, tomorrow, next year, next generation. The verbalism or rigid formalism of the earlier physiology and anatomy is of little value. Factual information is significant for the specialist but health attitudes, health habits, and health

* Read before the General Science Section of the Central Association of Science and Mathematics Teachers, Dec. 1, 1933.

ideals are of most worth for the layman or woman. If general science can induce in the high school student the firm belief that sickness can be reduced—that each person can enjoy better health—that science has methods of preventing the transmission of disease—that physical defects can be corrected—that rest, air, sleep, food, may contribute tremendously to the happiness, longevity, and vitality of men and women. Knowledge must be tinged and fired with feeling to mean much in the lives of men—without this feeling health information is like a dead fire.

It is not going to help a pupil much to be able to name the various kinds of teeth in his jaws if he neglects those teeth shamefully; there will be little value to the pupil in his being able to make an excellent diagram of the eye with a learned exposition concerning lenses, sclerotic coats, retinas, optic nerves, etc., if he reads in inadequate or glaring light, if ne neglects eyestrain, if he does not have an ideal of physical fitness, a desire for physical fitness, and certain fundamental habits that will contribute to physical fitness. How important is it that the student be able to name each of the 206 bones in his body if his habitually incorrect posture causes some of these bones to be deformed? What will it profit the high school student to gain a world of health information if he loses his own health?

General science should make for worthy home membership. Science problems arise in connection with lighting, heating, ventilating, cooking, cleaning, refrigeration, housing, clothing, and shelter. The boy or girl who can be led to see the place that science plays in making the home more efficient and more comfortable will really come to comprehend what writers of science textbooks mean when they write somewhat as follows: "General science can give the high school student a rational, orderly understanding of his environment so that he may correctly interpret that environment and that he may, to a considerable extent, be master of it. This approach to the problems of science through the inductive study of familiar, simple phenomena is sound pedagogy in that it proceeds from fields that are known to the less familiar and the more remote."

General science can aid our students in developing the problem-solving attitude if we habituate them to the scientific procedure and imbue them with a keen admiration for this method. The following quotation from Dewey bears strikingly upon this point of view. He says: "The future of our civilization depends

on the widening spread and deepening hold of the scientific habit of mind."

This technique of science should dwell upon the significance of problem solving—the sensing of problems in various environmental situations—the collection and use of materials to be used in solving the problem—the formulation of an hypothesis or possible solution of the problem—the testing of this hypothesis and its acceptance or rejection—the further testing of this hypothesis (now called a theory)—finally its irrefutable proof and its formulation as a law or principle. Here, it seems to me, is one place where our science teaching has not been so effective as it should be. Our students do not understand thoroughly the distinction between these terms. They do not comprehend that an hypothesis is an explanation believed to be correct but not proved; that a theory is an hypothesis the truth of which has been strengthened by experiment; that a law or principle theory proved to be correct. Our students can be trained to defer judgment until all the facts are in. They can, in other words, be led to appreciate the methods of science which have been developed and used by creative workers. In too many instances our courses in science have not generated in our students the power and the habit of scientific thinking. Professor Downing, of the University of Chicago, cites a test of thousands of high school students which indicates that those students with much science are no more proficient in this particular than are those with little science. It may be argued that science instruction is not effective in producing proper techniques in students. I will grant the argument, but I still maintain that *correct* science instruction will produce students able to employ the tested procedures of science. The point I am endeavoring to clinch here is that science instruction will produce scientific thinkers only if both teacher and pupil are consciously seeking that end. It is another illustration of the fact that the proverb—*Practice makes perfect*—is true only if the practice is *correct practice*. The values inherent in science instruction will attain fruition only if they are intelligently and persistently worked for.

Science can contribute vitally to the recreation of mankind. There are students who can be trained to get more pleasure out of popular readings in science, in books, and magazines than they can derive from reading in the field of belles lettres. Not all students, it is true, but some. Teachers of science can not rely upon the teachers of English to take care of this task for them.

Teachers of English are quite properly, because of their predisposition and training, more interested in poetry, essays, novels, and dramas than in expositions of the marvels of science in the biological or physical fields. Teachers of science are neglecting a great opportunity to develop enthusiasm for science and a means whereby the student can enjoy his leisure time if they fail to develop the habit of reading extensively and understandingly about animals, plants, machines, chemicals, diseases and their prevention, and kindred topics. There is a tremendously fascinating field here for teachers of science to explore and develop. It is not enough that our students, at our behest, laboriously dig out material for themes. They must be trained to enjoy reading of this type and they must become so habituated to it that they will voluntarily continue such reading as a form of recreation.

Another form of leisure-time activity which science can foster is the cultivation of some hobby. Science can aid the student develop a deep and abiding interest in trees, or flowers, or birds, in the radio, or the camera, or the engine, or in the reactions of various chemicals. Too much do we Americans sit as passive auditors or spectators and, in a sense, challenge the moving picture, or the athletic team, or the stage play to amuse and entertain us. We should realize that the keenest enjoyment comes to us from mental and physical activity of our own. Care of plants or animals, making and use of various types of machines, can provide a decidedly worth-while use of leisure and can enable the individual to derive a wholesome enjoyment out of life. Again it must be remarked that this recreational value of science can not be realized unless such value is held clearly in the minds of teacher and pupils from the outset.

Science can help our students comprehend man's power to control to his own ends the habits and relationships of animals and plants and the various forces. Our students can be led to see that man's bitter struggle for existence has been lightened by the applications of science on the farm, in the factory, and in the mine. Our science instruction should train our students to do understandingly such work as has to be done. Bridges must be constructed; buildings erected; roads laid; trains, automobiles, and airplanes manufactured; food raised; clothing manufactured. How well the needs of mankind are met depends upon how extensively and thoroughly science and its applications are comprehended. Food, clothing, shelter, communica-

tion, transportation—the sources of all the vocations of mankind are dependent upon science. Our students can be trained to explore themselves and the world about them. They can sense the needs of the world as well as their own abilities, aptitudes, and interests. They can choose their vocations with some degree of intelligence when they know science lies back of each vocation.

In connection with the vocational significance of science instruction it may be well to consider its potential influence upon citizenship. Many people feel that material progress has outstripped social conscience in the affairs of mankind. Such persons point out the anomalous condition existing in recent years wherein our captains of finance and industry assured us that the panic was the result of over-production—too much wheat, these gentlemen said, too many shoes, too many homes, too many automobiles, etc., yet simultaneously everyone knew that millions lacked all of these things and wanted them and were willing to work for them. We haven't produced too much—our methods of distributing the wealth produced by the applications of science has broken down. We need to apply scientific methods to social welfare. Our students must be trained so thoroughly in scientific modes of procedure that they will approach social problems with a scientific attitude of mind. Science should train our students so they will weigh and evaluate every proposal and will cease blindly following blind leaders.

Another aspect of the citizenship value of science is in its emphasis upon community health and sanitation. Prevention of disease, sewage disposal, water supply, building regulations, and kindred activities are all dependent upon science.

HUGE STELLAR LOOKING-GLASS WILL TAKE TWO YEARS TO COMPLETE

A huge looking-glass for the stars which will eventually serve science as the 80-inch diameter mirror of the new McDonald Observatory, in Texas, is now slowly cooling in the Corning Glass Works.

Its temperature is dropping two to four degrees a day from the high heat that the molten glass had when it was poured. In about three months the huge disc, a foot thick, nearly seven feet in diameter, weight 5,600 pounds, will be ready for shipment to Cleveland where experts of Warner & Swasey Co. will spend from one to two years grinding its face into a very precise concave optical shape.

McDonald Observatory, whose chief telescope will have this 80-inch mirror, is being erected on Mt. Locke in the Davis Mountains of Texas. It will be operated jointly by the University of Texas and the University of Chicago.—*Science Service.*

MATHEMATICS AND REALITY*

By Charles S. Slichter
Dean of the Graduate School, University of Wisconsin

Everyone has known since the days of the Greeks that the approach to reality is not through the senses alone, however realistic sense perception may seem to us. The world brought to the threshhold of our senses seems indeed a very real world, but even in our infancy we face some rude contradictions. It seems absurd, for example, that we infants cannot reach and touch the moon as well as reach and touch our rattle. It is a fortunate experience that we begin life by crying for the unattainable and by learning emphatically that our powers of apprehension are limited. But later we study theoretical mechanics and there learn how to reach and touch the moon and how to reach and touch many of the other early unattainables by the arm of reason. So in order to become grown-ups and not remain merely infants, we learn the technique of a twin approach to reality—the approach through the senses and the approach through the power of reasoning.

Science, they say, depends upon measurement, and hence, they assert, that all the ideas in the world of science are derived from the world of sense perception. Nevertheless, as Max Planck has remarked, common sense is not alone sufficient, there is a place for reason, and not only a place, but a position of supreme control. To illustrate this, I ask you to think of an imaginary scientist who spends all his daylight hours in his laboratory with his instruments of brass and glass and all manner of gadgets, making and recording a multitude of observations. At night, let us suppose, this same scientist spends his hours in his study, mathematizing in the midst of his manuscripts and books—studying and thinking in terms of the postulates and symbols of mathematics. Let us suppose that the scientist continues indefinitely; day after day, observing in his laboratory, and night after night mathematizing in his study. Suddenly it dawns upon him that there exists a correspondence between the observations of the day and the mathematical meditations of the night; more than that, he finds that the mathematizing of the night fills in great gaps in the experiences

* Read at the General Session of the Annual Meeting of the Central Association of Science and Mathematics Teachers, December 2, 1933.

of the day; more than that, he finds that the studies of the night extrapolate or add truth onto the end of the facts of the day; more than that, he finds that meaning and consistency and system and an aesthetic and satisfying living unity is given to the drudgeries of the day by the mathematical ecstacies of the night. Now let us ask at what time is the scientist in closest approach to the world of reality;—when recording and averaging the imperfect readings of the instruments of the day or when dreaming the mathematical visions of the night? The answer to this question is the test of mathematics. Nevertheless, sooner or later, after years or it may be after many generations, it is inevitable that the mathematics of the night will no longer check and coordinate the observations of the day. Such an event marks the beginning of a new epoch in science, for only as contradictions and paradoxes arise are the postulates and devices of the mathematics then used subjected to the revisions and generalizations and extensions that a new view of truth requires. Of necessity, to finite minds these periods of revolution must often recur and be repeated indefinitely. Sufficient only unto the day are the mathematics thereof. Paradoxes and contradictions to the scientist are but the foundations upon which to plan and structure new truth.

A scientist should be the humblest of men. He soon learns that he dwells not in a world of reality but in a make-believe world. Reality is forever beyond him. He slowly moves towards it but it ever eludes him. As Max Planck has said: We are compelled to contemplate the nature of things through spectacles of whose optical properties we are entirely ignorant and whose elements of design must remain unknown to us forever.

Even though science by its very nature is an artificially built up and make-believe world, this does not mean that it does not direct our gaze toward truth and reality, and of course it does not mean that mathematics is not the most potent guide to that end.

What is the nature of this guide that we call mathematics? There is no need to define it, for all creatures higher than the brutes are born mathematicians. People mathematize constantly. All games that we play are merely examples of mathematics. Even golf is mathematics, although we admit that it is not pure mathematics, for to the definitions and postulates and conventions of golf have been added many special and personal and materialistic and profane attributes that have no

place in science. A better illustration is the game of contract bridge. This is a two dimensional set up, North and South, East and West. It is played with fifty-two symbols in four suits and among the postulates is one that spades are greater than hearts and diamonds are greater than clubs, and so forth, and among the conventions it is said that West follows South in order of play—in clockwise rotation. Of course, it need not be played with material symbols nor by material players. It can all be perfectly symbolized in print in a corner of the Sunday Supplement and all of its orderly processes described for the reader. You may object to calling bridge a clear example of mathematics because of all the uncertainties that are involved. These uncertainties mean of course that it is only a richer sort of mathematics than that of the elementary school book type. After all, there is not as much uncertainty as you think. I always know, for example, that my partner will make the very worst play possible—there is no uncertainty about that. But there is a real objection to my illustration, because mathematics, like all science, must be productive and creative and the game of bridge as played is set up to be the same day after day and night after night. We are not even permitted to say that tonight we will play in reverse order and that East shall follow South, and so on; we cannot even suggest for this particular occasion that the Queen shall be greater than the King, even though the simplest concession to gallantry requires it, and we can prove that nothing new would result from this commutation. Bridge is not truly mathematics until it is made the subject of a doctor's thesis. This appears to be the infallible test of science. Such an opportunity can readily be provided, however. For a thesis, let the student study the domain of a *three dimensional* game of bridge, played by six players, North, South, East, West, Up and Down, played with a pack of seventy-eight cards in six suits, three red and three black, and the order of play shall be, for example, South, Up, West, North, Down, East, and so forth. You object at once that there are physical difficulties in arranging the table and players, especially the up and down players, to fit into the system. But all these difficulties have nothing to do with mathematics, which has no concern with matters of physical support or of materialism in any form, or, in this case, with the difficulties brought about by the force of gravity. It is alone required that the three dimensional game be completely and adequately symbolized in the thesis and that

every play and score be conformed accurately to the conventions and postulates set forth on page one of that thesis.

The proposed thesis illustrates the paradox that all games are mathematics but, nevertheless, mathematics is not a game. It is necessary to emphasize the essential expansive and constructive and creative and ungamelike character of mathematics. Mathematics never ceases its generalizing and enlarging processes. The writer of the doctor's thesis on the three dimensional game of bridge, later in life will undoubtedly write a paper for a mathematical journal on the game of contract bridge of n dimensions, played by $2n$ players with a pack of $26n$ cards divided into $2n$ suits of 13 cards each, n black and n red. Also later on, if the writer of the paper is truly industrious, he will attain the Nobel Prize in mathematics by his ingenious exposition of the game of bridge of an infinite number of dimensions played by a doubly infinite number of players with a pack of cards of an equal number of suits. For this game, he will probably postulate and set forth a rate of play so dense that a game may be finished in a finite time, less than a number of seconds itself less than E, and the players and card tables defined so comfortably small that the locus of each game but defines a single molecular cubicle in the space of the mathematician's imagination.

One of the most serious afflictions of the human race is the inborn and violently hereditary deformity of right-handedness. I do not mean right-handedness in the trivial sense in which man reaches with his right arm for food and drink, but I refer to that monstrous form of right-handedness by which man reaches for conclusions with the right arm of prejudice. He has developed the long and over-muscled right arm of prejudice, apparently to become his main help in time of trouble, and has constantly exercised it to his undoing. He still possesses, after ages of experience, only a short, and underdeveloped and underexercised left arm of reason, which nature intended to be the chief implement with which to reach for conclusions. I judge from conditions in Europe at the present moment that humanity at this hour is as strongly right-handed as ever. Conclusions are mostly reached by the long arm of prejudice. Woe to Europe and woe to all of us if we do not remember that it is written: "If thy right hand offend thee, cut it off!"

The left arm of reason is constructive and peaceful; the right arm of prejudice is destructive and combatitive in its uses.

From this arises responsibilities to all teachers of mathematics. Here, in mathematics, is the one domain, the one insulated island of refuge, where the left arm of reason can be freely exercised and developed. Here youth can at least learn that there exist domains of truth where prejudice and destruction have no place. How important it is, then, that all youth without exception be made familiar with this domain. Especially should the training in elementary geometry be made and kept universal and studied and mastered by all. It is no accident that for centuries Euclid has been studied by the residents of the Inns of Court who expect to be called to the British bar. Geometry at least shows to youth that there exists one region where the left arm of reason is powerful and wherein its muscles and reach can be tested and synchronized. He will better appraise the devastations of prejudice if for a time he has dwelt where it is non-existent.

In my lifetime there have been two developments in the power and authority of mathematics that are important enough to be called characteristics of the age in which we live. One of these is the amazing fact that all natural science has become mathematical, so rapidly in fact that for the first time we are taken back to the doctrine of Pythagoras, who made the first outspoken claim for the place that abstract thought must hold in solving the mysteries of phenomena. Pythagoras was a supergenius. He saw that the comprehension of the world about him was to be sought in the revelation given by mathematics. In an outburst of intuition he proclaimed that "the nature of things consists in number." In the present age, for the first time, this marvelous dictum is fully realized. The adequate answer to any question in any science is now just a number. It is no longer even the red or blue qualitative test of acidity of the litmus paper—it is the "hydrogen ion concentration" expressed as a number. We can echo the words of Blake who said "Go deep enough and there is music everywhere" by the dictum, "Go deep enough and there is mathematics everywhere." The various natural sciences have become but phases of mathematics. Physics is the clanking noisy part of mathematics; chemistry is the smelly part of mathematics; biology is the mussy part of mathematics.

Along with the development just described, there has been another important change, namely, the making of elementary mathematical instruction optional and elective. This last tend-

ency is playing havoc with sound education and is in direct contradiction to the needs of the age. It contributes to the further development of the right-handedness of the race, at a time when rapidity of communication and other modern developments have but added more danger and more explosives to an already over-prejudiced age. The leadership in elementary education is not sound at this point. Youth needs to dwell for a brief spell on the island where he can at least learn that his left arm of reason exists and has use. Ambassador Bryce said, "It is the duty of the schools to reflect the spirit of the age without yielding to it." We should hold for adolescent youth at all hazards the possibility for adventure in the use of his left arm of reason and not yield in this respect to the spirit of the age. We cannot approach reality through the reachings of prejudice.

The scientist does not claim that he possesses the only approach to reality. He realizes all too well that he can only view reality through spectacles whose optical properties are unknown. He is willing to admit that the poet and the mystic also command powers of vision and indeed often view reality through spectacles whose optical properties are more perfect than those of the scientist. The scientist is all too conscious of the unknown abberations and distortions and crossing of rays that are inherent in his spectacles. He admits that the poet often visions reality with less distortion, with more uncrossing of the rays and more direct parallelism in the lines of sight than is possible in the make-believe world of science. The scientist is aware of the artificiality of the domain in which he works—he knows that his postulates and imaginings, and set-ups and changeful theories are indeed just make-believe—quite as make-believe in fact as in the game of bridge. He envies the poet his more direct and often more inspired approach. "Go deep enough and there is music everywhere." This saying of Blake goes to the root of things as no dictum of science possibly can. It is indeed hard to believe;—it implies so much. It means that if we go deep enough, there is beauty everywhere; it means that if we go deep enough there is goodness everywhere; it means that if we go deep enough there is harmony everywhere. On the surface of things we see prejudice and ugliness and pain and suffering and wickedness, but Blake would say "Go deeper, go deep enough into the nature of things and there is music everywhere." Who would deny, or at least, who would wish to deny that Blake is right; for it is indeed a vast symphony that is being

scored, although only the initial dissonances have as yet been written down, and ages and ages must elapse before the opening theme is fully announced. Blake would have us believe that a Great Presence is mindful of the orbits of life; that there always exists a best way and many less perfect ways and that the Great Presence would guide and shield the orbits of life in the ways that are best and lead them more and more to a perfect purpose. These words make up the Creed of Evolution and they hold alike for galaxies and for men. Go deep, go far into the scheme of things, and there is mathematics everywhere. Go deeper, go nearer and nearer to the core of reality, and there is music everywhere.

HIGH SCHOOL INDUSTRIAL CHEMISTRY*

By E. G. Pierce

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In describing our course in technical chemistry, I have no missionary zeal whatever. I have no wish to induce anyone, anywhere, to attempt developing such a course. Probably there are only a few schools located in the midst of chemical industries in which such a course would be advisable, yet there seems to be much interest in high school technical chemistry if we may judge from the number of inquiries about it which we receive.

It has required many years of effort to fit our course to the needs and capacities of the students, to the apparatus and materials available, to our educational objectives, and to the needs of Cleveland industries. There has been much trying and discarding, which must still continue in order to keep up with constantly changing conditions.

Ours is not a trade course. That was one of the first ideas to be discarded. Our primary purpose is to educate rather than to turn out chemical workers or to supply the direct needs of any particular industries. Our effort is to capitalize the urge and enthusiasm for chemistry which many boys have, as a means of leading them to think. But students must have definite things about which to think. Why not permit them to think about the things in which they really have an interest, and to think about the facts, materials, laws, theories, and concepts which will

* Read before the Chemistry Section of the Central Association of Science and Mathematics Teachers, Dec. 1, 1933.

become the working tools of their profession rather than learning to think about laws, theories and information which they may never use?

Technology would certainly not be the best method for general education but it does seem a splendid thing that, in our modern diversified school systems, it is no longer necessary to force all students through the same groove regardless of their individual interests, talents, and desires.

The technical courses in our school are not intended to take the place of college instruction. But not all students can go to college. Is it to be supposed that the accident of birth in financially favored families automatically selects those best fitted to take advantage of advanced training?

In any city of nearly a million people there will always be a considerable group of boys who will be prevented by lack of funds from going directly to college, but who are just as talented, just as keen and eager, and often far more industrious than those who are able to go. The public technical schools are a great boon to such students, giving them a start in the lines in which they are interested. Most of these boys do arrive at college some two or three years later. They are then more mature, get more useable value out of college life, and find it much easier to fit themselves into industrial life because of their practical experience.

The matter of employment was one of our greatest difficulties at first, because employers found it difficult to believe that a high school boy could really know anything about the more intricate phases of laboratory procedure or production work. That difficulty has been overcome and now the larger industries have definite places for both the technically trained high school boy and the college man.

Another difficulty we have always encountered is that few seem able to comprehend the large amount of time actually devoted to our technical courses. When the boy spends half a day every day for two years on one subject he becomes fairly saturated with the lore, theories, skills and practice of his chosen work. It is only by means of this unusual allotment of time, in my opinion, that technical chemistry may successfully be taught. Students in our school may take a combined technical and college preparatory course so that they will be prepared either way. Many boys have never been able to continue their education except in extension courses and in the school of ex-

perience, but have arrived in responsible positions regardless.

We have not been able to develop any definite laboratory work in chemical engineering projects, partly because of lack of equipment, also because the analytical laboratory seems to be the main opportunity for our graduates. We do, however, attempt some individual projects with individual students as opportunity may offer.

Probably the outstanding improvement in our analytical procedure has been the substitution of the rapid routine methods of industrial laboratories for the more academic methods usually described in quantitative texts. These, in the hands of high school boys, give greater accuracy and a wider variety of experience than by trying to follow an intricate maze of minor refinements in method.

Organic theory and laboratory work are offered during the last semester, not with the idea of giving anything like a complete survey of the subject, but to turn the tide of student thought and interest into the channels of synthesis and production following the three semesters of analytical work. Petroleum, sugar, rayon and rubber industries are studied thoroughly.

The subject matter and methods for class room work have called for more experimentation than any other part of the course. All that is new in the chemical world comes up for discussion, but we are constantly forced back to the conclusion that fundamentals should still be the chief food for high school boys. Since employment is at least a secondary objective, one should not forget that employers do not pay off boys for what they know about positrons, electrons, or the quantum theory. The best preparation for college chemistry also is a thorough grounding in fundamentals. Serious instruction in many of the more advanced phases of chemistry should, in my opinion, be left to the colleges and more effort in high school devoted to establishing faith, in, and accurate knowledge of the fundamentals. Employers in industrial laboratories want boys to follow instructions faithfully, to make analytical calculations accurately, to keep a precise record, to make an intelligible report, to carry on a number of analyses simultaneously with neatness and speed, and to do honest work. Those are excellent qualifications for college or any line of endeavor.

Our industrial projects may be of more general interest than other phases of the course. These may best be described

by presenting several narratives of boys and their projects.

Dr. Earle R. Caley, whose many articles you may have read in various journals of the American Chemical Society, and who is now a Professor of Chemistry at Princeton University, graduated at East Tech. from our department in 1917. As a student he became much interested in collecting gravimetric factors. His new book, "Analytical Factors and Their Logarithms," evidently was begun at that time.

C. J. Pew '19, constructed a class room micro-projector which we have used ever since. The reflector is a dental mirror mounted on a shaft made from a wire nail. This work was done about five years before such a device appeared on the market. Mr. Pew worked at the Arco Paint Co. for several years, then went to Ohio State Univ. where he took the Chemical Engineering course, and continued as an assistant in analytical chemistry, taking a Master's degree. He is now employed in the U. S. Forest Products Laboratory and is working for a Doctor's degree at the Univ. of Wisconsin.

John Hannum '27, while studying organic chemistry conceived the idea of tracing the synthesis of every type of organic compound from the elements. The chart which he constructed is still a most valuable contribution to our equipment. Mr. Hannum has since graduated from Case School of Applied Science and expects to complete the requirements for the Doctor's degree at the Univ. of Michigan this year.

Chet Horton '21, was an expert welder while still in high school. Becoming interested in metallography, he built some of the equipment we still use. He followed up welding as a profession, and is now a welding engineer with the Oxweld Acetylene Co., in which capacity he designed and constructed part of that fine welding exhibit at the Century of Progress. He has continued the project which he began with us by building a complete metallographic outfit and a number of pieces of laboratory apparatus which have helped him attain his present position. Being unable to attend college, he worked out a complete program of advanced study which he followed as his work permits.

Henry Poole '28, became interested in metallurgy by being allowed to work out a project on the extraction of metals by the thermite reaction. He graduated as president of his class at Case School of Applied Science, and is now working at the Univ. of Idaho on a scholarship.

Another organic chemistry project which is popular with all

our students is the collecting and exhibiting of their preparations. They seal some of the products in medicine vials and mount these on cards with a legend about the materials and their production.

While studying industrial chemistry, many of the boys construct models of the machinery and processes in that line. A model of a Blake jaw crusher just made by Joe Chelenko '34, is actuated by rubber bands and works perfectly.

Alex Vass '33, became deeply interested in the analytical balance during his second semester and has constructed a device which seems so original, we are taking steps to protect his idea. To quote from his note-book, "The Electro-Magnetic Balance consists essentially of an analytical balance with the addition of an air-core electro-magnet having a traction variable from 1500 to 10 mg. This magnet is placed under the right-hand pan of the balance and accomplishes the same purpose as would the known weights; therefore the weights lighter than one gram are eliminated. The strength of the magnet is varied by means of suitable resistances which cut current from the circuit until the weight of the object on the left pan is just enough to pull the right pan away from the magnet. The weight of the object is then read directly from the vernier dial attached to the variable resistance."

These rather rambling descriptions of different phases of our work attempt to give an impression of the course rather than any continuous or complete details.

SOME PHYSICAL UNITS

By Ross McConnehey

Not all physics teachers are on speaking terms with the following units. Are you?

mole	therm	bel	electron volt
slug	hertz	lumen	maxwell
farad	lambert	phot	radian
gauss	diopter	micron	faraday
mil	abvolt	gilbert	barye
poise	joule	roentgen	oersted
lux	bar	mho	angstrom
henry	hefner	poundal	dyne

Common sense in an uncommon degree is what the world calls wisdom. Coleridge.

HIGH VACUUM*

By M. N. States
Central Scientific Company, Chicago, Illinois

The history of vacua, which covers a period of approximately 300 years, is a very fascinating study. Notwithstanding this long historical background, in common with many scientific developments, the important and the significant advances in vacuum technique have been made with increasing intensity during the last half century. In recent years, the introduction of high vacuum phenomena into the various phases of modern life is so extensive that vacua have important influence on nearly every scientific problem, whether the problem is one of pure research or of industry. The flavor of our breakfast coffee is reputed to depend upon the vacuous state of the packed container. I am told—once each week if I listen—that the flavor of a certain brand of coffee results from its being packed in containers under atmospheric pressure. On the other hand, the distributors of another brand declare that coffee must be packed in vacuum-tight cans under reduced pressure if it is to be palatable and if one may enjoy a second cup without ill effects. It is not my intention to enter into a discussion of this controversial question; I mention it merely to indicate the wide spread influence of vacua in modern life.

With this brief reference to the history of vacua and to the far-reaching influence of vacua in modern affairs, we shall pass on to the subject matter of this afternoon's paper. A brief synopsis of the contents of the paper may serve as a guide to you as well as to the speaker.

In the first place, I shall hazard an answer to the question, "What is a high vacuum?" We shall discuss some recent experiments dealing with high vacuum technique, referring to new, or to relatively new, methods for the production of high vacua. In this connection, we shall discuss methods for the measurement of vacua, as well as certain factors associated with vacuum pumps, which either limit the usefulness of the pumps or enhance their value for certain experiments. By making only brief reference to certain published accounts of recent experiments it will be possible for me to report at some length on certain

* Read before the Physics Section of the Central Association of Science and Mathematics Teachers, Dec. 1, 1933.

experimental results on this subject which have not been published.

"What is high vacuum?" In 1922 Dushman (1) states, "The pressure which we ordinarily deal with in high vacuum phenomena range from 1 to 10^{-3} bar* and even less." In the same paper he says, "The highest vacua attainable at present range around 10^{-4} bar." Dushman (2) in a 1931 paper states, "It may be stated as a fundamental requirement of a high vacuum system that it shall be capable of reducing the pressure to about 10^{-3} microns of mercury." Hickman (3) in a more recent paper says, "The greatest reduction of pressure yet secured is 10^{8} molecules per cubic centimeter, and here every device known to the physicist was invoked." If we translate these statements into a common language, namely millimicrons ($m\mu$) of mercury, we have

Dushman, 1922—High vacuum range	750 to 0.75	$m\mu$
Highest attained vacuum	0.075	$m\mu$
Dushman, 1931—Requirement for high vacuum system	0.75	$m\mu$
Hickman, 1932—Highest attained vacuum	0.003	$m\mu$

From these figures published by men who are recognized authorities in the field of high vacua we may draw at least three important conclusions. First, we may regard phenomena which occur under pressure conditions of one micron or less as high vacuum phenomena. While this dividing line between vacuum and high vacuum is somewhat arbitrary, we shall show later that there is good reason for making such a division. Second, marked progress has been made in recent years toward attaining the ultimate goal in vacuum technique—the perfect vacuum. At best, however, Nature has a score of one hundred million molecules per cubic centimeter in her "abhorrence of a vacuum." Third, the data show a marked improvement in high vacuum pumps and high vacuum technique during the past ten years.

It is difficult for one to get a proper perspective of the molecular state of matter in a high vacuum and to coordinate this with conditions at atmospheric pressure. There are at least two reasons for this: First, the numbers required in quantitatively expressing the conditions are either so large or so small that they are quite meaningless. Second, the actual change is much larger than we realize. I venture an illustration in an effort to elucidate the relationship.

* The bar as used here is the c.g.s. absolute unit of pressure, which is a dyne per square centimeter.

The micron is a millionth of a meter. It is commonly known as a thousandth of a millimeter. A millimicron is a thousandth of a micron, or a thousandth of a millionth of a meter. Just how large is such a length? The unit of length which Professor Michelson used in the measurement of the standard meter was 643.84696mμ and he showed that this unit of length was contained in the meter 1,553,163.5 times. Light of shorter wave length than 400 mμ is invisible to most eyes. Hence, objects having dimensions of the order of 200 mμ or less cannot be rendered visible by any type of instrument. However, there are many ultra-microscopic organisms smaller than 200 mμ. In fact, it has been shown that certain mosaic plant disease viruses may be as small as 10 mμ.

Suppose that one such ultra-microscopic virus is clinging to the wall of a vacuum system. At atmospheric pressure the virus, assuming its surface area to be 100 sq. mμ, would be hit 3×10^{11} times every second by the air molecules. When the pressure falls to one micron of mercury the number of hits per second is 4×10^{5}. A further reduction of the pressure to a millimicron and the highest attained vacuum decreases the number of hits per second on the virus to 400 and to 1, respectively.

Let us now consider briefly the technique of measuring high vacua. Undoubtedly the best known, and perhaps the most widely used manometer for high vacua is the McLeod gage. In the hands of one who has a proper understanding of its limitations and who is familiar with the technique of conditioning a McLeod gage, it is a useful vacuum accessory. Were it not for the fact that the McLeod gage is finding increased favor in certain quarters, and at the same time, is so soundly abused in other quarters, I would not refer to it this afternoon as it can scarcely be classed as a recent addition to the vacuum set-up. One authority (4) says, "The McLeod gage is still the standard instrument for the practical measurement of low pressures. Nevertheless, its accuracy is quite uncertain below 0.1 micron." It would be easy to find others who concur in this opinion. Furthermore, the opinion is commonly held that the indication of a McLeod gage will *always* be higher—that is, a higher pressure indication—if a freezing trap is used in connection with the gage. Stated the other way around, they maintain that the gage reads too low when the freezing trap is not used. In general this may be true, especially if the gage is frequently opened to the atmosphere. However, if a low pressure is maintained within

the gage at all times, it frequently happens that when a refrigerant is applied to the trap the gage will show a lower pressure.

The McLeod gage shown in Fig. 1 has functioned satisfactorily in our laboratory almost daily for a number of years. The

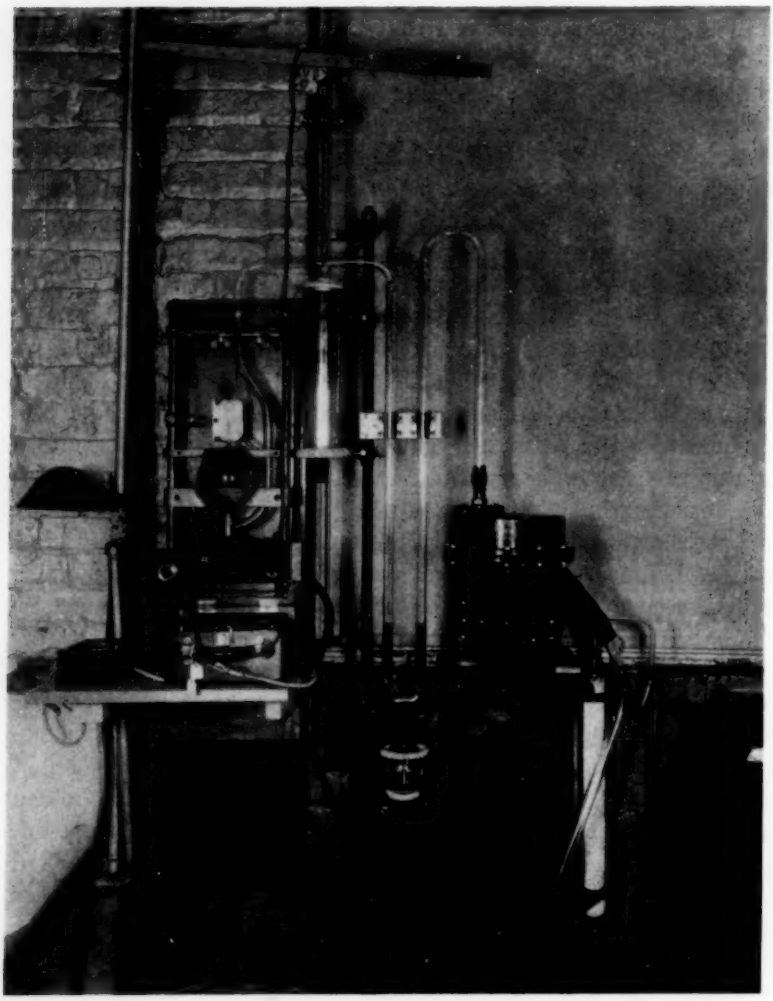

FIG. 1. Arrangement of apparatus for testing "Aristovac" pumps, including a McLeod gage, ionization gage, mercury cut-off, and pump.

gage has a constant of 1.77 when the pressure is expressed in millimicrons and the gage readings are expressed in millimeters. This means that the gage reading is approximately 1.33 mm for

a pressure of 1 mμ and 13.3 mm for 100. mμ. The capillary has a diameter of about 0.8 mm. The bulb has a capacity of more than 1 liter, and the gage requires 20 kilograms of mercury for its operation. The ionization manometer (shown in the upper part of the figure and referred to in a later experiment) was calibrated by means of this McLeod gage in the range 600 to 20 mμ, and over this range a linear relation between the readings was obtained. At 1 mμ pressure, the McLeod gage agreed with the ionization manometer within the limit of accuracy of reading the length of the mercury column. Thus the reliability of the gage for pressures less than .1 μ is greater than is commonly believed.

The gage is used in checking molecular pumps, one of which is shown connected to the gage, and the apparatus is constructed so that pumps may be interchanged without admitting air to the main vacuum system. The pressure in the gage rarely, if ever, reaches 1 μ, inasmuch as the fore pressure in the molecular pump is usually of the order of 10 μ before the trap is opened. When not in use, but when the pumps are operating, the entire gage is kept hot by means of electrically heated resistance wire (not shown in the figure) wound around the glass parts. Occasionally, all glass parts of the gage are thoroughly heated by means of a gas flame as a final step in the conditioning process. Liquid air or dry ice is used on the trap when measurements are being made.

The ionization manometer originally described by Buckley (5) and improved by Found and Dushman (6), and others, is justly finding increased favor in high vacuum technique. The gage is essentially a three-electrode tube having a filament, a grid and a plate. In the operation of the manometer the grid functions as the anode while the plate or the collector, as it is called, is held at a small negative potential with respect to the cathode. With this arrangement the electron current flows to the grid, producing ionization in the residual gas. The ions thus produced are drawn to the collector and form what is called the positive ion current. Within certain limiting values of currents, of applied potentials, and of pressures, such a device when connected to a vacuum system obeys the following relation:

$$p = C \cdot i_p / i_e \qquad (1)$$

where, p is the gas pressure, i_p is the positive ion current i_e is, the electron current and C is a proportionality factor.

If the electron current is maintained constant, Eq. 1 reduces to

$$p = K \cdot i_p \qquad (2)$$

where, $K = C/i_e$. This relation is valid over a considerable range of pressures.

As the device is not an absolute manometer, it must be calibrated against the known indications of another gage. A McLeod gage is used for this purpose. In this calibration the electron current is held constant and the positive ion current is plotted against the readings of the McLeod gage. If the component parts of the tube—cathode, anode and collector—are properly arranged, a straight line relation obtains between the gages for pressures below 1 μ. The fact that the calibration curve is not the same for all gases is a disadvantage. However, after the gage constant has been determined for a particular gas, pressures below the range of the McLeod gage may be determined by extrapolation. This extrapolation seems to be limited primarily by the leakage current of the tube. This leakage current may be as small as 10^{-10} amperes, which, in a recently constructed gage, corresponds to a pressure of 5×10^{-4} mμ of mercury.

Jaycox and Weinhart (7) of the Bell Laboratories have recently described an improved ionization manometer. They found the constant of the gage to be valid over a wide range of pressures below 4 mμ of mercury. They state, "This ionization manometer is easy to construct, can be used to measure pressures of the order of 10^{-3} mμ of mercury, and has a long life in properly sealed off tubes."

The Knudsen absolute manometer, the many variations of this molecular gage, the Pirani, and the thermocouple gages have been carefully studied by various experimenters. For reasons into which we can not go, these gages are not so widely used as the two mentioned above and we shall therefore not consider them at all this afternoon.

A moment ago, in establishing the limits of the high vacuum range, the statement was made that there is good reason for the division at about 1 μ in addition to the mere opinion of the investigators. It is obvious that those who have made such a division did so for other reasons than their ability to obtain a certain vacuum. We shall now consider this question in some detail, emphasizing its connection with the operating characteristics of vacuum pumps.

In 1909 Knudsen (8) derived an expression for the flow of gas through a tube of length L and radius R, which equation is valid for any pressure. This expression, which is semi-empirical, has been found to fit experimental facts in a remarkable way. The equation may be written

$$Q=\left(aP+b\,\frac{1+\alpha P}{1+\beta P}\right)(p_1-p_2) \tag{3}$$

where,

$$a=(\pi R^4)/(8\mu L)$$

$$b=\frac{\sqrt{2\pi}}{\sqrt{\rho}(3L/4R^3+2/R^2)}$$

$$\alpha=(2\sqrt{\rho R})/\mu$$

$$\beta=(2.47\sqrt{\rho R})/\mu.$$

In addition to the above defining equations for a, b, α and β, Q is the quantity of gas which passes through the tube per second measured at the pressure P; ρ is the density of the gas at unit pressure; μ is the coefficient of viscosity of the gas; (p_1-p_2) is the pressure difference between the ends of the tube.

The first term of the equation, $aP(p_1-p_2)$, fits the condition of hydrodynamic flow and is the dominant factor at high pressures. The second term is characteristic of molecular flow and is the controlling factor at low pressures. The constants a, b, α and β, are functions of the dimensions of the tube, the viscosity of the gas and its density at unit pressure. The effect of the length of the tube enters into the first term of b—the term containing the factor L/R^3. The fact that the tube has two openings is taken care of by the second term in b. It can be shown that for any given set of conditions there is a pressure for which Q is minimum. This critical pressure is a function of the dimensions of the tube and the nature of the gas, but it is of the order of a few microns for most vacuum systems.

Fig. 2 is a graph of Eq. 3 for a tube having a length of 10 cm and a diameter of 1 cm, through which air at 20°C. is flowing under unit difference in pressure. The quantity of gas is measured in cubic centimeters at unit pressure; that is, the ordinate is $Q/(p_1-p_2)$. P is the average pressure existing between the ends of the tube measured in microns of mercury. The curves are plotted to the same scale, but the one representing the hydrody-

namic flow has its vertical scale displaced relative to the other two curves as shown on the right. The effect of each term is obvious. The pressure corresponding to minimum flow is about 3 μ. That the molecular flow is the dominant factor even at pressures greater than 3 μ is clear. Knudsen has shown that molecular flow accounts for 95 percent of the quantity of gas delivered through such a tube, if the pressure is less than a value where the ratio of the radius of the tube to the mean free path of the gas is 0.2. Hence, we may define the condition for a high vacuum by saying that it is the state where the flow of gas is molecular in character. From the above analysis, the requirement or the

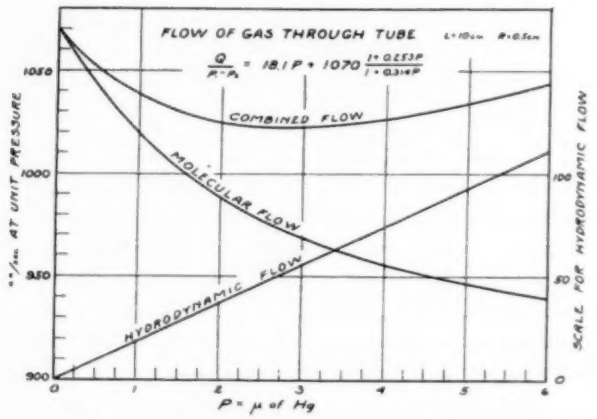

FIG. 2. Curves showing the relative importance of hydrodynamic flow and molecular flow of gas through a tube at low pressures.

specification for a high vacuum will depend on the particular vacuum system, but modern vacuum systems are generally constructed so as to make this definition agree with that previously given.

When the condition of molecular flow obtains, the value of Q is given by Eq. 4.

$$Q = b(p_1 - p_2). \tag{4}$$

Since this is the condition which obtains in the high vacuum system a further analysis is of importance. In the first place Eq. 4 is analogous to Ohm's law for electrical circuits. We may interpret the quantity $(p_1 - p_2)$ as a difference in potential and the quantity Q as a current, in which case b corresponds to a conductance or, in the language of alternating currents, to an admittance.

To compute the effect of several tubes, joined end to end and having different lengths and cross-sections, one sums a series of terms each of which is the reciprocal of a quantity b. Thus

$$1/Z = 1/b_1 + 1/b_2 + \cdots 1/b_n \tag{5}$$

where, Z is the resultant conductance of the tubes. Such procedure is analogous to method for finding the conductance of a number of electrical conductors connected in series.

The magnitude of Z is of extreme importance where speed in the evacuation of a system is essential, or where large volumes are to be evacuated, as in the case of a modern vacuum spectrograph or of a large x-ray tube. The advantage of using a vacuum pump of large capacity is lost if the tubes which join the pump to the system are not properly selected. On the other hand, pumps of small capacity can be made more effective by the proper choice of connections.

Recently, Mr. Bond (9) made an interesting study of the relationship between electric circuit and molecular flow theory. He found that the theorems of electric circuit theory could be advantageously applied to a vacuum system in which a tube corresponds to admittance or conductance and a volume to capacitance. Only an abstract of the paper has been published, but the complete account of the experiments is on file in the library of the Department of Physics, University of Chicago. The theoretical analysis and the corroborative experimental results are a real contribution to molecular flow theory.

To link the molecular flow of gas through a tube with a vacuum pump in a quantitative way, it will be necessary to consider the speed of a pump. In 1913 Gaede (10) defined the speed of a vacuum pump by means of Eq. 6.

$$\frac{dp}{dt} = -S/V \cdot (p - p_0). \tag{6}$$

In this equation dp/dt is the rate of the reduction of the pressure in a volume V at the time t and when the pressure in the system is p. p_0 is the limiting pressure of the pump and system, and Gaede defined S as the speed of the pump. If one solves this differential equation there results

$$(t_2 - t_1) = \frac{2.3V}{S}[\log_{10}(p_1 - p_0) - \log_{10}(p_2 - p_0)] \tag{7}$$

From either equation one obtains a definition of the speed of a pump. For example, from Eq. 6, the speed of a pump, S, is the volume of gas which the pump can remove from a sufficiently large enclosure in one second, when the pressure in the enclosure is p and the volume is measured at the pressure $(p-p_0)$.

It is important to distinguish between the speed of evacuation of a system and the speed of a vacuum pump. If the speed of a pump is determined by direct application of Eq. 7, the value of S so obtained is the pumping speed, and not the speed of the pump. It is a function of the speed of the pump and the conductance of the connecting tube. However, if the conductance of the system is known, the speed of the pump may be determined as the speed of the pump, S_1, is related to the actual pumping speed for the pump and system, S, and the conductance of the system, Z, by

$$1/S = 1/S_1 + 1/Z. \tag{8}$$

Ordinarily apparatus for the measurement of the speed of a pump is so arranged as to make Z very large, and hence $1/Z$ negligible, or easily obtainable from calculation. As an example of the influence of the conductance of a tube on pumping speed, the apparent speed of a pump having a capacity of 5000 cc per second, connected to an enclosure by means of the tube whose conductance we considered—a tube 10 cm long and 1 cm in diameter—will be reduced to 800 cc per second. Obviously a fast pump may have its speed effectively reduced by the use of improper connections.

Mechanical molecular pumps, introduced by Gaede (11) in 1912, have recently been improved and are finding a place in high vacuum technique. Gaede's original pump, under optimum operating conditions, had a speed of 1400 cc/sec and produced a vacuum of the order of 1 mμ. Holweck, several years later, designed a molecular pump which corrected some of the inherent defects in the Gaede pump. The Holweck pump has a slightly greater speed but the limiting pressures of the two pumps are the same. Siegbahn has developed a molecular pump which consists of a disk rotating between grooved plates, by means of which gas is forced from the periphery of the disk to the axis, where it is removed by the fore pump. Both stationary disks are grooved so that the pump action is the same on either side of the rotating disk, i.e., from the periphery toward the axis. The limiting pressure of this pump is 10 mμ.

During the past two or three years a molecular pump possessing certain novel features has been developed in our laboratory. Several names have been associated with the pump in the course of the development, but the official name is the "Aristovac." A vertical cross-section of the pump is shown in Fig. 3. The disk, which rotates some 10,000 R.P.M., is attached directly to the shaft of the motor. The motor is enclosed within the pump hous-

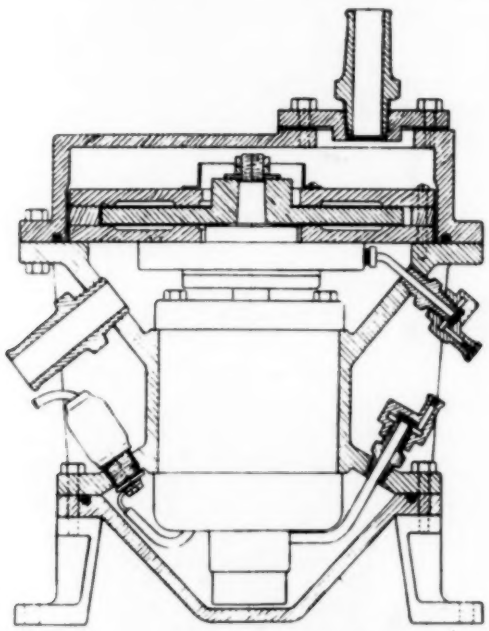

FIG. 3. Vertical cross-sectional view of the "Aristovac" pump.

ing and operates under the reduced pressure maintained by the fore pump. The outlet to the fore pump is shown at the left, while air enters the pump through the port in the dome. The disk, which is carefully machined and ground, rotates between upper and lower stationary, grooved plates. The plates, showing the arrangement of the grooves, are illustrated in Fig. 4. The clearance between the disk and the stationary plates is not more than 0.05 mm. This necessitates the careful fitting of all parts of the pump mechanism, and the rotor—the disk and armature—must be statically and dynamically balanced.

The pump operates most efficiently when the pressure of the vacuum system has reached a value where the mean free

path of the molecules is very large as compared with the clearances between the moving disk and the stationary plates. That is, the pump is essentially a high vacuum pump and operates best when molecular flow conditions obtain. At such a pressure,

FIG. 4. An "exploded" view of the "Aristovac" pump.

or for any lower pressure, the molecules collide, more or less alternately, with the disk and stationary plates. The interchanges in momentum which occur between the surfaces and the molecules, force the air along the groove in the upper plate to the edge of the disk, down over the edge of the disk and then radially through the groove in the lower plate toward the axis

of rotation. The air then passes through the openings in the upper bearing assembly into the fore vacuum space in the bottom of the pump housing. This action is so effective that the ratio of the pressure in the fore vacuum space to the pressure in the dome of the pump is 10^4 or greater. Hence with a suitable fore pump, the pressure in the vacuum system may be reduced to about 0.1 mμ of mercury.

A diagramatic arrangement of apparatus is shown in Fig. 5 by means of which the speed of the pump was determined. The procedure was as follows: Initially the 22 liter volume was evacuated until the limiting pressure was determined. This was found to be approximately 2 mμ. Dry air was then admitted to

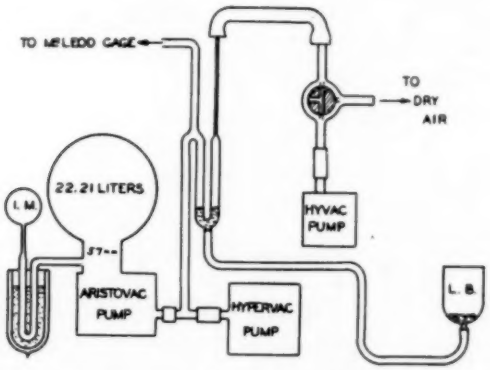

FIG. 5. Arrangement of apparatus for measuring the pumping speed of "Aristovac" pumps.

the enclosure through the mercury trap which was connected between the fore pump and the Aristovac pump. When the pressure within the enclosure reached some desired value, as indicated by the ionization gage, the trap was closed and the pump, running all the while at full speed, quickly evacuated the 22 liter volume. Pressure—time readings were made as the evacuation progressed, and the data were plotted on semi-logarithmic paper as shown in Fig. 6.

The graphs in Fig. 6 are representative of many trials and clearly show that the speed of the pump is independent of the initial pressure, p_1. The connection between the pump and the enclosure was such as to make the value of $1/Z$ in Eq. 8 negligible as compared with $1/S_1$, and therefore the speed of pump was determined directly from Eq. 7. The fact that straight lines were obtained show that the speed of the pump is constant

over a wide range of pressures. The full data show that the speed is constant for pressures ranging between 1μ and 1 mμ. The slope of the lines yield a value of 1600 cc per second for the speed of the pump.

The broken line in Fig. 6 indicates a decrease in the speed of the pump resulting from increased speed of rotation of the disk. It was found that either higher or lower speeds of rotation of the disk reduced the speed of the pump.

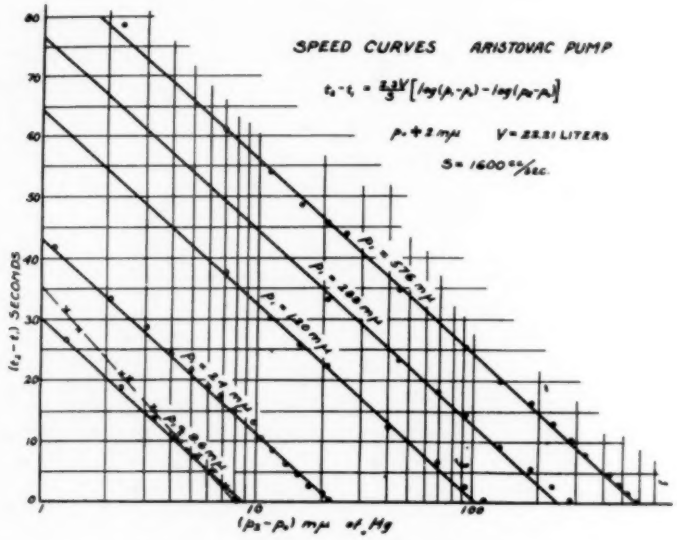
SPEED CURVES ARISTOVAC PUMP
V = 22.21 LITERS
S = 1600 cc/sec.
P_2 = 576 mμ
P_2 = 288 mμ
P_2 = 144 mμ
P_2 = 72 mμ
P_2 = 36 mμ
P_2 = 18 mμ
$(t_2 - t_1)$ SECONDS
$(P_2 - P_1)$ mμ of Hg

FIG. 6. Curves showing the constancy of pumping speed of the "Aristovac" pump over wide pressure ranges.

Recently interest in methods for obtaining high vacua without the use of a refrigerant has been marked. This is a difficult, if not an impossible, task where mercury or other substances which have high vapor pressures are in communication with the vacuum system. The result has been accomplished in a variety of ways, but the characteristics of the molecular pump are such as to make it easily adaptable to this requirement. A well outgassed charcoal trap at room temperature located between the Aristovac pump and the vacuum system is sufficient to prevent vapors from entering the system for many hours. In one such trial, a vacuum of 0.3 mμ of mercury was maintained for five days.

Mills (12), Hickman and Sanford (13), Becker and Jaycox

(14) and others have recently developed, or have experimented with, diffusion pumps in which organic liquids such as n-dibutyl-phthalate, butyl benzyl phthalate or such an oil as Apiezon "B" replace mercury. These liquids have very low vapor pressures and produce excellent results in well designed pumps. A charcoal trap at room temperatures is often sufficient to stop the pump vapors from entering the vacuum system. Such pumps have been constructed which develop speeds of from 7 to 10 liters per second. T. L. Ho (15) at the California Institute of Technology has published two papers recently on mercury diffusion pumps. These pumps are used in connection with high voltage x-ray tubes and one such pump, having nineteen nozzles, developed a speed of 321 liters per second. Time will not permit a further description of these pumps.

In closing I wish to express my appreciation for having this opportunity of speaking to the Physics Section of the Central Association of Science & Mathematics Teachers. Like the air in a vacuum system, the subject "High Vacuum" is inexhaustible, but I trust that I have referred to one or two points which may have interested you. Someone has said that a philosopher, in the process of his development, learns more and more about less and less until finally he knows everything about nothing. Not being able to qualify even as a philosopher in a state of development, it is not surprising that my information on the subject, which, by the above definition, should be the ultimate in philosophical investigations, is limited.

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GEOMETRY'S TRIBUTE TO TRADITION*

By Elizabeth B. Cowley
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The wholesome boys and girls who crowd into our geometry classrooms bring with them a refreshing joy of life and a curiosity to understand themselves, their future, and the world—the world which is to them so full of wonderful mysteries and treasures.

What has geometry to offer to these restless, mischievous, inquiring, speed-loving young people of to-day? In trying to answer this question we must keep in mind that if geometry is to gain the attention of these sophisticated young folks and retain their interest and their allegiance it must compete successfully with the movies, high-powered cars, airplanes, and cheap magazines filled with thrilling detective and mystery stories and tales of crime and adventure.

Nor is our problem confined to the adaptation of material to the needs and capacities of our pupils, for what geometry succeeds in offering in a school depends to a certain extent upon what might be called the administrative aspect of the subject. Progress may be assisted or thwarted (1) by a city or state course of study with its suggested or prescribed aims, content, and methods; (2) by the textbook in use; (3) by uniform examinations or batteries of tests given to the pupils.

We must also have information in regard to the person who sits in the teacher's chair. Is he worthy to be regarded professionally as a teacher of geometry? Is he active and alert; has he native intelligence, seriousness of purpose, and a thorough mathematical and general education? Does he teach, or is he merely a drill master who prides himself upon his success in coaching his pupils to obtain high ratings on recognized uniform examinations or certain standardized tests?

What geometry can offer that is suited to its present clientele, what the teacher is permitted or required to offer, and what the teacher wants to offer are, alas, not always in harmonious accord. The content and methods of presentation recommended by the school authorities, or the content and methods in actual use may lag far behind the best which is offered by the pioneers who are pushing ahead on the new frontiers.

* Read at the Mathematics Section of the Central Association of Science and Mathematics Teachers, Dec. 1, 1933.

How is tradition related to any or all of these questions? Geometry is generally regarded as an essentially traditional subject, and is often referred to as a useless study that tradition alone keeps in the high school curriculum. It is popularly conceded that everyone should know enough simple arithmetic to make change and pay his bills. It is argued that more complicated mathematical questions should be left to those queer folk who delight in the subject, giving financial matters to bankers and certified accountants and designing, building, and construction problems to architects and engineers.

It is not to be wondered at that there is objection to geometry, for the traditional geometry is supposed to be devoted wholly to such monstrosities as icosahedrons, orthocenters, pentadecagons, perigons, rhomboids, trapeziums, incommensurable magnitudes, isoperimetric polygons, mutually equiangular polygons, scalene triangles, ratios of similitude, reductio ad absurdum, and Q.E.D. Mischievous pupils have enlivened this assortment by adding such picturesque phrases as "the devil's coffin" and "steps to hades."

Who is to blame for this popular misconception of geometry? The unfamiliar and unnecessary names of Greek and Latin origin are not an essential part of the subject;—they are simply part of geometry's tribute to tradition. Euclid wrote in Greek, because it was his native language and it was spoken and understood by his hearers. Please note that Euclid did not try to mystify his readers by writing in Sanskrit.

The early English translators of the Latin translations of Euclid's Elements naturally used many words of Greek and Latin origin. Even though these words were seldom used outside of geometry, there was no objection to them then, for in those days students of geometry were well versed in the classics. It is amazing that there are to-day teachers, school administrators, writers of textbooks, and makers of examinations and tests who insist that geometry shall continue to pay this same tribute to tradition. These persons are certainly not facing the situation that exists in many classrooms, where not one pupil has ever studied either Latin or Greek, and where the pupils or their parents came from Slavic, Teutonic, or Celtic countries. It may seem incredible but many a teacher has found that pupils of Italian or Greek parentage often know nothing of the classics, and are timid in acknowledging even the slightest acquaintance with Italian or modern Greek. This curious state

of affairs seems to be due to their consuming zeal to become Americanized as rapidly as possible.

Another evidence of geometry's tribute to tradition is to be found in the slavish copying of the form of Euclid's immortal Elements. It has always seemed to me that if Euclid were to appear on the scene today, he would be filled with amazement and contempt for those who are adhering so closely to his book and revering it as absolutely perfect. Euclid solved his own problems. Present day teachers should solve theirs. His problem was to gather together scattered theorems and arrange them in an orderly and logical manner.

Euclid was writing for university students. Geometry continued to be a university subject for many centuries. Two hundred years ago Harvard students studied it in their senior year. It was not until the middle of the last century that Harvard required geometry for entrance. When this subject was regarded as wholly a college preparatory study, the methods of teaching it differed little from those that had been employed in the universities.

Conditions have changed rapidly since that time. The demand for public high schools arose and soon began to be insistent. Recently the high school population has not only increased with great rapidity, but has also changed enormously in character. In our public high schools today there are many pupils who never can go to the traditional college; and many of them never should go, for it would be a waste of their time and someone's money. Nevertheless, many of these pupils can and do profit from the study of geometry—provided it is presented in a way fitted to their needs. Incidentally, in adapting geometry to the needs of the non-college pupils, there is an opportunity to make it more vital to all pupils, including the college preparatory group.

Let us see in what other ways our problem to-day differs from Euclid's problem. Euclid's hearers in 300 B.C. were mature, leisurely Greeks—men who knew no algebra as we know algebra today, but who were skilled in logic and were fond of disputations. They despised the manual arts and disdained material ends. Moreover, Euclid's own status differed radically from that of a public high school teacher of to-day. In the earliest universities, such as Alexandria where Euclid was, there was no organization. Each teacher attracted his own followers and attended to his own matters of curriculum, housing, and

finance. He could teach exactly what he pleased—provided only that he could get anyone to listen to him.

Our American audience in A.D. 1933 is immature; it knows some algebra but little logic; it delights in the mechanical arts; it sets a high value on material ends; and speed is the breath of its life. We have compulsory school attendance. We teachers and pupils are parts of a highly organized machine. Moreover, each state has a certain jurisdiction over every educational institution within its borders, regardless of whether it is on a public or a private foundation.

It is no disparagement of Euclid's Elements to say that its form is not suited to the needs of the secondary school pupils of today. Is it really a tribute to the great geometrician to continue to use translations of his book in situations to which they are not adapted? Would it not be a more genuine tribute to Euclid to catch the spirit of the man and the book and clothe that magnificent spirit of elementary geometry in a modern dress, casting aside the outward form which is now outgrown? Gladly would we pay tribute to Euclid in such a spiritual sense.

Geometry has also lost power by paying tribute to the traditional method of teaching the subject. This method is to send the whole class to the blackboard to write out the formal proof of a theorem which they have learned verbatim from their textbooks and then have one boy recite the same proof orally to his thoroughly bored and intellectually passive classmates. The substitution of paper for the blackboard does not remove the objectional features from this method.

The story of any educational reform is always a tale of struggles against the brainless following of tradition. Let us cite a few examples of various attitudes toward changes suggested in attempting to adapt the traditional geometry to the needs of modern secondary school pupils.

(1) When a famous surgeon who also lectures in a university school of medicine first saw one of the recent progressive texts in geometry, he asked the author whether there was any real need of a new textbook in the subject, since the sum of the three angles of a triangle is still equal to two right angles, and the Theorem of Pythagoras continues to hold true. When the author asked whether his students used an anatomy textbook written several centuries ago, because the human body had not changed, he exclaimed, "You have given me something to think about."

(2) An attorney wrote to the mathematics master of a successful preparatory school saying, "I am taking the liberty of sending you a recent textbook in geometry. As I read it, I thought how much more geometry would have meant to me then and now if I had studied from such a book. I venture to say that this book would make geometry more valuable to my son." To which the master replied, "I taught you geometry in such a way that you passed College Entrance Examinations with 100%, and I feel confident that I can do as well by your son. I challenge anyone to get better results than I obtain. No one of my boys has failed in College Board Examinations in many a year. Last year no one made under 95%. However, I did look at the book you so kindly sent me. As soon as I saw that it used a positive definition of parallel lines and that it omitted Q.E.D., I knew instantly that I would not like it and that it would therefore be of no use in my classroom. Hence I am returning it with thanks."

(3) A man who was considered by a public school superintendent as an exceptionally able teacher of high school geometry said at a conference of teachers of the subject, "It strikes me that all this talk about improved methods of teaching geometry is punk. The correct procedure is to get a good old-fashioned textbook free from fads and frills, and compel the pupils to memorize the vocabulary, definitions, and proofs in full. We can not do better than to follow the grand old traditions without variation. Who are we that we should try to improve upon Euclid?"

Let us analyze these cases. The surgeon and the attorney show the open-minded attitude of intelligent laity and their willingness to listen to the teacher or the textbook writer who is facing the geometry problems courageously and trying to find solutions. The mathematics master and the high school teacher show that the teaching profession still contains members of the "old guard,"—persons who accept tradition blindly, instead of thinking things out for themselves. Both these teachers exhibit that terror of a new order common to all persons who have vested interests in the old order and are either unwilling or unable to make the necessary adjustments for a new order. They fear variation, because it might upset a fine piece of mechanism of which they are parts. Each teacher has an exaggerated idea of the value of his own services, because each knows only a part of the large questions involved.

It would do the whole educational world an immense amount of good if such men were suddenly forced to change to wholly new and unfamiliar surroundings. Either they would learn much and learn it rapidly, or else they would give up trying to teach. The leisurely and fastidious master with his small classes of superior boys from cultivated homes should be suddenly taken away from the beautiful buildings set in a spacious suburban estate and placed in a noisy, crowded public high school in a smoky industrial district where the classes are large and the pupils exhibit great variety in native ability, in home and racial backgrounds, in ambition, and even in such personal matters as courtesy and cleanliness. Would he survive? If he did, what would happen to the pupils? The high school man might be awakened if he were suddenly transferred to a small private school of the so-called progressive sort, where the pupils are encouraged to ask questions freely—politely to be sure—but quite candidly and frankly. How would he reply to their searching questions? Either of these teachers could be described as a foreigner characterized another man, "The trouble with that man was that he always got his emphásis in the wrong places."

Capable teachers who are mathematicians have made real progress in adapting geometry to the needs and capacities of secondary school pupils of today. It is exhilarating to see the way in which progressive teachers and textbooks approach this old, old subject from an unsuspected angle and actually transform the cheerless drabness of the traditional text into a bright and cheerful book so brimful of the things that are vital that the would-be hater of geometry becomes, in spite of himself, thoroughly interested in the subject and works as only an interested pupil can work.

These teachers and textbooks are carrying forward the movements begun long ago. We Americans think of two publications of a decade ago which not only formulated the ideas of the past, but also gave inspiration for further progress. I refer, of course to the publications of the National Committee and the College Entrance Examination Board. Their lists of theorems and the list of the Regents of the State of New York rendered valuable service, especially in two highly commendable features: (1) simplification of the technical vocabulary, and (2) reduction of the number of theorems by the elimination of many that are trivial.

We must not assume that the United States is the only coun-

try in which these questions are being studied. Synchronous with our National Committee's Report was the first edition of the Report on the Teaching of Geometry in Schools, prepared for the British Mathematical Association. We all know something of the Perry Movement, which aroused British teachers of mathematics at the beginning of the present century. A quarter of a century ago Professor Dr. Felix Klein, the internationally known Professor of Mathematics at the University in Göttingen (Germany), was trying to impress upon the mathematical world the necessity for reform in the teaching of mathematics in the secondary schools.

As alert and thoughtful teachers have been experimenting and devoting intelligent study to such adaptations of the content and methods of presentation as will better meet the needs of the pupils, they have long realized that there are definite causes for the high rate of mortality in beginning classes in demonstrative geometry. Let us turn our attention to some of these difficulties which have been perpetuated by geometry's blind tribute to tradition.

What does the Introduction of the traditional text include? (1) A list of names and dates of Greek mathematicians and the theorems attributed to them—all of which is meaningless to the beginner. (2) Pages of formal definitions, beginning with such undefinable terms as point and line, and including many that the National Committee recommended to be abandoned. (3) Definitions of axioms and the so-called postulates as "self evident" truths. (4) A half dozen curious drawings that might lead the unobservant to incorrect conclusions. These drawings are given for the purpose of convincing the pupil of the necessity for formal demonstration. But they are a failure. The dull pupil is merely perplexed by them and his able classmate is amused, especially if the first theorem given formal demonstration seems to him to be as "self evident" as the axioms.

How much more vital an Introduction would be; (1) if the Greek mathematicians were replaced by live facts about geometry today, and much simple construction work requiring easy drawings with compasses and a straight edge; (2) if we abandoned the attempt to define the undefinable and omitted obsolete and unnecessary terms and if the formal definitions of the necessary words were preceded by informal explanations and followed by simple exercises that require the pupil to use the word in various situations; and (3) if the expression "self evi-

dent" were omitted, and the pupil were told that there are statements in mathematics which we assume, or take for granted, and that there are other statements which we are not willing to accept unless we have thought about them until we are convinced of their truth. This process of thought that convinces us is called a proof. At first we write down our thoughts informally. But we soon find that if we arrange our proof in a certain form, we can save our time and effort and increase our accuracy and add to our own ease and that of others who read our proofs. Thus we have a formal proof, or demonstration. With this explanation, the Introduction should give many informal proofs and a careful explanation, step by step, of the theorem that is to be Theorem I. By this method the pupil will acquire a sense of the continuity of geometric material and he will not look upon axioms and theorems proved informally as outcasts from polite geometric society. It is surely evident that the first theorem should be one whose proof is straightforward and entirely free from quibbles.

The use of theoretical superposition at the beginning of Book I has cut short the geometrical career of many a hopeful and promising boy. The lad who survives this monstrosity may be slain by the too early introduction of indirect proofs or by the lack of unity in the methods of proof in the early theorems—say the first ten. Indirect proof has a legitimate place in a course in geometry, but it is not suitable in the beginning of the course. Nor are the pitfalls confined to the Introduction and Book I. We shall mention only one that comes later—namely, the traditional treatment of ratio and proportion, with its dots, its antecedents and consequents, its alternation and inversion, its composition and division, and its if a is to b as c is to d, then *Q.E.D.*, ad absurdum. The subject could be made so clear and simple if a ratio were treated as a fraction and proportion as a fractional equation, because the pupils have studied fractions and fractional equations.

The blind followers of tradition insist that any new arrangement of any part of geometry would confuse the beginner. This is absurd, because the beginner does not know the tradition. The person who would become confused is the teacher who knows the old order but either can not or will not bestir himself to learn a new arrangement. How inappropriate it is that a person who loves ease and dreads activity, change, and adventure should be placed as a teacher of active boys and girls of high

school age! Many who follow tradition too implicitly fear that any of these suggested adaptations would inevitably lower the standards and lead to the so-called soft pedagogy. This is an error. Any boy will tell you that it is easier to memorize a proof printed in the text than it is to work out part or all of the proof, or to think how the theorem can be used in exercises, especially in those dealing with everyday work and play. In his own vernacular he will characterize the progressive teacher as "tough," and of his classroom he will probably say, "Its lots of fun there, but, oh boy, you've got to keep awake and get busy."

It is natural and wise to scrutinize carefully every suggested variation, for we must ever be on our guard against the things that are merely passing fads and the changes whose only recommendation is that they are novel and startling. In regard to any suggested change we may well ask three questions;—(1) Is it offered by someone who has acquired a deep and broad mathematical background by prolonged and discriminating study of many branches of the subject? (2) Is it offered as the result of careful and patient experimentation in classrooms? (3) Is it offered as a remedy for a difficulty which is definite and is generally recognized? If each of these questions can be answered in the affirmative, is not the suggestion worthy of careful consideration? The teacher or author who has struggled valiantly to find remedies for the recognized difficulties welcomes fair discussion. The person who fears discussion is the man who has blindly accepted inflexible tradition.

To sum up;—The facts of geometry are eternal and changeless and its spirit is calm and serene, but the character of expression, the transient manifestations of these facts and this spirit are forever in a state of flux, ceaselessly changing to meet the varying conditions in our physical and social worlds. New needs require new techniques.

The modern teacher or writer of a textbook in geometry must choose with genius and fidelity the finest latent parts of the subject and so adapt and fashion them to the needs of our modern times that his work shall express with beauty and candor a new concept of geometry which shall be vital and dynamic.

If the power to do hard work is not talent it is the best possible substitute for it.—GARFIELD.

A TEACHER'S APPROACH TO THE PROBLEMS IN TEACHING ELEMENTARY SCIENCE*

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It is necessary for us to seriously consider many problems if we are to promote and expand science instruction in the elementary schools. Certainly all of us are desirous of promoting such a program and make as few mistakes as possible. At the outset let us agree that science instruction in the elementary schools is not something new. It is one of our earliest forms of science instruction. Many of our best science teachers have made their greatest contributions in this field. Many of the difficulties in present day science teaching would have been eliminated if the fundamental principles these early teachers espoused had not been forgotten. One illustration is a statement we hear so often now—"We no longer need the laboratory."

In approaching the problem of teaching science in the elementary schools, the teacher's greatest problem is the development of a basic philosophy of science teaching. It is not enough to read some one else's philosophy and then try to believe it. It must be a fundamental part of the teachers being and thinking. How is the teacher to build up such a philosophy? Certainly not by saying nothing has been done, or forgetting what has been done by our early teachers. The first step, then, in building this philosophy is the acquisition of the knowledge of what has been done. This does not mean merely the knowledge of historical facts but the understanding of the ideals and thoughts of these early teachers of science.

What were the conditions under which these early teachers of science labored? Let us consider as an example, Agassiz and his Penikese School which was first opened in 1873. Possibly we can see why Agassiz and other teachers taught science as they did if we care to examine the conditions of living at that time. The country was largely a rural or agricultural nation. We had not recovered from the effects of the Civil War. There was no telephone, no electric light, no gas engine, no wireless telegraph, no radio, no automobile and no roads as we know them now. Homes were lighted by candles and were heated by

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fireplaces and stoves, if heated at all. Electricity as we know it now did not exist for the masses. There were no science books for children, or at least a very few. There was no laboratory equipment. Schools as we know them now did not exist. The life of the children and people at that time was so different from our present method of living that it is difficult for us to comprehend the conditions and difficulties under which they lived.

Under these conditions was it not the reasonable thing for these science teachers to take the pupils outside of school to study the objects of nature or bring these objects of nature into the school room? They were the only things they could use. They were also laboring under the tradition—"do not teach in schools the things pupils will learn anyway." At present we refer to such things as practical things. Is it not likely that in their eagerness to teach the things they could observe and handle they were apt to lose sight of the less tangible things which we now call physical science? You must remember the physical sciences were not as well developed then as now. All the equipment the pupils needed was a pair of good eyes plus the ability to interpret what they observed. But you can not teach physical science with that equipment. However, we should not get the impression that physical science was not taught during this period.

Now what happened to nature study from 1873 to 1920 or 1925? In this same period of time the physical sciences advanced by leaps and bounds. They developed more in this period than in all history, if we consider only the number of new inventions. These sciences have modified very materially our methods of living. Practically all of our modern conveniences are a result of this development. There were also a large number of developments in the biological sciences, but they have not had as much effect in changing our methods of living. Did our nature study teachers change the curricula and subject matter to meet these changed conditions? I believe you will agree with me they did not. As is usually done these teachers developed subject matter and methods for their life time. They did not continually sift and experiment to discover the subject matter and methods which would make it possible for boys and girls to adapt themselves to their changing environment. So in 1920 or 1925 we find many of our nature study teachers teaching the same subject matter as was taught in 1873 and very much in the same way, but with much less enthusiasm. This is not a criticism of our

early science teachers nor of the present teachers. I mention these facts about nature study to emphasize the necessity of studying the past if we are to develop a sane philosophy of science teaching and also to emphasize the necessity of continually revising this philosophy to meet changing conditions. It still further emphasizes the necessity of not permitting a philosophy to become static. It must grow. But this does not mean a teacher must change her philosophy with every changing fad. She should only change it as a result of evidence and experiences gained by herself, other teachers and investigators.

The history of the teaching of nature study gives us many valuable helps for our present day science teaching. It also teaches us that unless philosophies, methods and subject matter in science are continually changed to meet changing conditions the study will sooner or later lose its value. Let us not, however, get the impression that nature study is the only field of science instruction that has failed to adapt itself to changing conditions. This same criticism is made for all of our college sciences and for physics and chemistry in high school. The same criticism was made for botany, zoology and physiography a few years ago. The introduction of general science and biology has eliminated some of these criticisms, but there is still much room for improvement in both of these subjects.

A knowledge of the past is not sufficient in itself to develop a philosophy. Teachers should have a knowledge and understanding of the present conditions and methods of teaching. They should know what our present teachers are thinking about. What their problems are and how they are trying to solve them. What subject matter they are teaching, how are they teaching it and for what purposes they are teaching? How can we find out what teachers are thinking and doing? It seems to me the only way is to go to the teachers themselves and let them tell us what they are doing. The State Science Committee used this procedure in developing the "Wisconsin Philosophy of Science Teaching." Fifty-five groups and over three hundred teachers participated in the discussions. The members of the committee attended a large number of these meetings to listen to the discussions, but never to express their opinions. None of the teachers knew what the committee thought about developing such a philosophy. As Chairman of the Committee I was firmly convinced, as were the other members of the Committee, that its function was to interpret the teachers thoughts and

ways of doing things in terms of the present needs of science, and not to impose upon teachers the opinions of the committee. To my notion that is the function of any committee. If I were to make a criticism of the report of the Science Committee in the 31st yearbook of the National Society For The Study Of Education, I would say that it is dominated entirely too much by one school of thought and is not representative of the real thinking of a large majority of our teachers.

After a year's discussion of science teaching with these groups and a careful analysis of a great number of reports and discussions, the Science Committee proceeded to place in written form an interpretation of these reports and discussions. We spent from five to eight hours a day for three weeks at the task. We were determined that the philosophy should be brief and should be understandable to the average teacher. The meaning of every word we used was checked with the meaning given in the dictionary. Most teachers, we think, should be able to read this philosophy and interpret it in the way in which it was written. We also believe it is representative of what teachers are thinking in Wisconsin. I mention the method we used in Wisconsin because I believe that all groups of science teachers should place in written form their philosophies of science teaching as a means of interpreting their thoughts and actions. A large number of teachers told the Committee that in attempting to write their individual philosophies they really began to understand the meaning of science teaching. The personal accounts of the satisfaction many teachers received in doing well what they thought they could not do makes an interesting story.

What does the writing of this philosophy by teachers and the interpretation of past philosophies mean in terms of the present movement in elementary science as proposed in the 31st yearbook? In the first place, I believe, the Committee misinterpreted the nature study movement. In the second place it is built up largely on one school of psychology. In the third place it is questionable whether the development of the notion of large generalizations and large principles is practical and possible of attainment by children in the lower grades. The report does present a point of view and a program and it is the teachers task to evaluate such a program in terms of what has happened in the past, what teachers are doing and thinking now, and in terms of future progress. The report would be of more value to teachers if it had been developed as a means of helping

teachers interpret what other teachers are doing and thinking, and not so much as a means of promulgating one special school of thought, based on one school of psychology to develop one particular kind of teaching.

The next step in the development of a philosophy of science teaching is more visionary. The science teacher should look ahead into the future and build "scientific air castles." Are we willing to guess what the pupils we are teaching now will be doing when they have reached our age? What will our trains, airplanes and automobiles look like 20 or 30 years from now? Can we even guess what the radio will be like? What will television be like? How will we be living? Yes, and how will science be taught? I feel we need a philosophy which is forward looking and which will help us interpret the future. We will make many mistakes in predicting what the future has in store for us, but this is one situation where it is better to have tried and failed than not to have tried at all.

After having set up a philosophy of science teaching by science teachers, the next step in the approach to the problem of teaching elementary science is the selection and organization of subject matter. We have heard so much about subject matter and done so little. A great many methods have been used to select subject matter. We have pooled the opinions of textbook writers; we have analyzed courses of studies by different groups and committees; we have forced children to tell us they were interested in a great many things they never heard of, and we have asked adults and parents, who have been out of school at least twenty-five years, to tell us what we should teach. Imagine people going to a business man who has been out of business twenty-five years and asking him what modern business needs. Then, too, we have tried to get principles from a job analysis of the future occupations of children. Of course each child knows what he is going to do. The height of absurdity was reached in an article I recently read in which the author stated that girls in the junior high school should become interested in the future work of their prospective husbands and prepare themselves to help their husbands. It is the first time I have ever heard that girls in the junior high school knew who their husbands were going to be. It shows to what absurdities job analysis may carry us.

I might go on and enumerate other methods used to select subject matter, but the sad part of the whole procedure is—the

child has been forgotten. Subject matter as a rule is organized from the point of view of the adult. We think it must be logically organized and we forget that children learn largely by the trial and error method. After the children have been exposed to these logically organized courses for a few years, we expect them to understand their environment, to have a good scientific attitude and to understand the scientific method. We expect more from these children than the human race, as a whole, has learned in all history.

You are, no doubt, asking by this time, How will you select subject matter? I think the best method to select subject matter is through the careful observation of the activities and experiences of children by trained teachers. And you may ask, What do you mean by activities? I mean the things children do. What do children do? I believe we can include most of the things they do in ten categories or divisions. These categories are: (1) eating of foods, (2) sleep and rest, (3) play and entertainment, (4) work, household duties, chores, etc., (5) clothing themselves, (6) shelter, their home, (7) talking, (8) reading, (9) moving about—going places, and (10) health and personal sanitation. If we analyze these activities we will find the subject matter children need. For example, on the basis of such an analysis we find the principles which occur most frequently in play activities are gravity, friction and inertia. Many of the principles we have been teaching occur only infrequently on the basis of such an analysis. The subject matter selected in this way should be tried out at different grade levels and with different groups to discover where the material works satisfactorily. This is one of our greatest problems and will not be solved in a short time.

As far as possible, subject matter should be selected that is capable of being demonstrated, or performed as an experiment by the pupils, something that the children "can get the feel of." Something that becomes a part of their very being. We do not need to worry about complete understanding of some science principle if it becomes a part of a child's experience. If a boy learns that his wagon will tip over if he turns a corner sharply, he has a good beginning for the understanding of inertia. Simple experiments in the classroom will demonstrate that a plant needs light, water, food and air to grow, but it is not necessary for a child to know all about the complicated processes in plant growth to understand this. Who does understand plant growth? Simple experiments with the electric train

will give boys and girls some understanding of circuits and the elementary principles of electricity, but they do not need to understand the electron theory to play with an electric train. These illustrations are given to show that a great many science principles can be made a part of the experiences, reactions and feelings of children at an early age and they will understand them well enough to know how to use them. These experiences will give reason and some understanding to the things they do. As a contrast to this type of activity I mention the activity carried on by some teachers of having the pupils calculate how long it will take a train going so many miles per hour to travel from the earth to the sun. It is done, I suppose, to help pupils get some notion of distance, but how do we know it helps. I have always insisted that the task of getting a train to the sun is much more complicated than visualizing the distance.

I believe, then, we can get subject matter if we analyze carefully the activities of children. We will find many interesting principles to teach them. We can make these principles a part of their beings and experiences if we will only select the things they come in contact with. We can widen their experiences by bringing to each child what other children do. One school may share its experiences with other schools. But we must not let one type of activity over-shadow the others and make the same mistakes we have made before.

You are no doubt wondering how teachers can get the knowledge needed for the understanding of children's activities. There is no short-cut way of doing it. From my experiences with teachers of elementary science, I have found they worry more about subject matter than other phases of teaching. Many of them have been forced to teach elementary science without being adequately prepared to do it, but this is no fault of theirs. A teacher should have a basic understanding of the fundamental principles of science and an appreciation of what science can offer if she hopes to succeed. Much of the necessary knowledge can not be obtained from books, but must be obtained by coming in contact with real things and observing carefully what happens under different situations. The teacher must also have a basic understanding of the fundamentals of experimentation. How are problems solved on an experimental basis? What are experimental factors? What are controls and variables? If a pupil asks a simple question which is answerable by means of simple experiments, then the teacher should know how to guide

and direct the pupil to get his answer by experimenting. Some will say this means a teacher must have a scientific attitude. It means more than that, it means a teacher must be able to put this attitude into practice and also have an understanding of the scientific method. The necessary knowledge and understanding of the fundamentals of experimentation can be acquired by the average teacher if she is genuinely interested in doing it. It is not an impossible task, nor is it too much to expect that a teacher do it if she expects to become a real teacher of science.

The third and final step in this approach to the problems of teaching elementary science is the realization on the part of the teacher that some method of teaching is necessary. Teachers have improved wonderfully the technique of teaching in reading, arithmetic and language. Is it too much to expect them to do the same thing in science? Does it mean a good teacher of reading and arithmetic will also be a good teacher of science? I do not believe we are prepared to say they will at the present time. However, I know of nothing that will prevent them from becoming good teachers of science. What training will help these teachers to become good science teachers? It does not mean that they be able to use the method of some other teachers, but the acquisition of a method of their own. It means a method which teachers can use in the situations in which they find themselves and which is adapted to their personalities. No doubt our teacher training teachers have a real service to perform here, but I admit I have not found the best way of doing it. It will do us no good to say that teachers are born and not made, nor will it help individual teachers to become outstanding teachers by forcing all of them to adopt one pattern of teaching even though that pattern may be good. It does not take long for such a pattern to become formalized to which science itself is a protest. *This is what happened to nature study.*

I have heard educators say that if a teacher has a basic philosophy of teaching and is grounded in subject matter, methods will take care of themselves. This may be true, to some degree in college, but it certainly is not true in the elementary school. To me method means putting into practice the ideals and philosophies of the teacher. Subject matter is the tool. A teacher must know subject matter and know how to use it in terms of her philosophy if she is to be a good teacher. Teacher training institutions can help teachers acquire this subject matter, they can help them acquire a philosophy of science teaching, they

can demonstrate to them methods of science teaching which are or have been successful, but they can not teach for them. The teacher must realize that it is necessary for her to develop her own methods of teaching if she hopes to be an outstanding teacher.

In discussing the teacher's approach to the problems of teaching elementary science, I have presented some barriers and guide posts for our thinking. There is going to be, it appears to me, a rapid expansion in science instruction for children. In order that teachers may approach the many problems in this field sanely and progressively I have said teachers need to develop a basic philosophy of science teaching which is based on the experiences of the past, and on present conditions. This philosophy is not to be used as a means of defending the status quo, but as a means of guiding teachers in thinking through future difficulties as they meet them and helping them to adapt themselves to changing conditions as they come.

I have also said teachers need to select subject matter from the activities and experiences of children and teach in such a way that it becomes a vital part of their lives and the life of the teacher. A good teacher will combine judiciously her philosophy, subject matter and methods.

I have not attempted to say anything new, but I have attempted to place emphasis on what I consider the most pressing problems which need to be solved if we are going to make rapid progress in this most difficult field of science instruction, science for children—or elementary science.

ONE ALASKAN GLACIER ADVANCES WHILE ALL OTHERS RETREAT

The only known case of an Alaskan glacier advancing was called to the attention of the Geological Society of America, by Dr. Chester K. Wentworth and Louis L. Ray of Washington University, St. Louis, Mo.

During the past fifteen or twenty years, Dr. Wentworth said, Alaskan glaciers generally have been melting back faster than they have been flowing forward. The net loss has been as much as 170 feet in some glaciers, as little as 20 feet in others. But the Taku glacier alone has been in rapid forward movement. It has advanced 7,600 feet since 1909.

The cause of this anomalous advance is a mystery. Dr. Wentworth suggested the possibility of an earthquake's having caused the dumping of a huge quantity of snow on the glacier, perhaps in 1899 or at some time since. But, he added, no wholly satisfactory explanation has been formulated.

—*Science Service*

BAKING AND CHEMISTRY TEACHING*

By Victor E. Marx
Staff Chief and Editor, "Bakers' Helper"
Chicago, Illinois

It is a conservative statement I make when I say that when one sits down to a meal, whether it be breakfast, luncheon, or dinner; whether it be week day, Sunday, or holiday; whether it be in the years of childhood, of manhood or of old age; there will be on the table one or more products of the bake oven. The importance of bread as a food is emphasized during times of catastrophes such as floods, fires, earthquakes, wars, and depressions, when bread forms the major portion of man's subsistence, at least temporarily. In times of great prosperity when people indulge themselves with the luxuries of life, some turn away from bread toward foods which are no more valuable from the nutrition standpoint but which suit their fancies and satisfy their whims.

Baked foods have become so much a part of man's life that they are even associated with religious ritual in practically every denomination. A momentary thought of the tradition of one's own religion will bring to mind some uses of bread, of wafers, of matzoths, of Hot Cross buns, or of some other form of baked product which has a religious significance. These thoughts I mention to impress you with the intimate connection baked products have with the lives of individuals. Is this not reason enough why high school teachers should be better acquainted with the baking industry, and impart to their students, at least a superficial knowledge of the fundamentals of baking?

Baking has become one of our major industries. It is the second largest food industry, being exceed in the value of its products by the meat packing industry only. It is the eighth largest industry in the country. Seldom do we think of baking as being in the same size group as steel, oil, automobiles, movies, and clothing. It employs close to a fifth of a million people. Bread alone, not including any other baked product, is produced in this country by commercial bakeries at the rate of 340 pounds per second, day and night, Sundays and holidays included.

Baking is really a chemical industry. The ingredients used in the making of baked products are those which are used in the

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home. These foods are manufactured under chemical control Such ingredients as flour, water, yeast, sugar, salt, butter, lard, vegetable shortenings, eggs, spices, milk, and fruits are common to all people, and are milled, purified, grown, refined, manufactured, churned, preserved, prepared, and produced, as the case may be, under strictly scientific conditions. The industries which supply both the baker and the consumer with these foods employ chemists to maintain uniformity of the products and to constantly study ways and means of improving them and developing more economical methods for producing them. Is it not, therefore, logical to expect that the baking industry which combines all of these good foods into baked products, should be very much a chemical industry?

Baking is not only a chemical industry, but depends very largely for its success upon the control of biological processes. In making a piece of furniture, or an automobile part, or an electrical device, every piece is machined with precision, and measurements of parts are held to the accuracy of thousandths of an inch. Bread cannot be made with such precision by such methods. No two loaves are identical in size, in every detail of shape, or even in appearance. This is because the making of bread is a living process. One readily agrees that no two radishes, or two cucumbers, or two potatoes are identical in size, in shape, in appearance, or even in flavor. The *apparent* uniformity of bread loaves, is a measure of the skill of the baker, and proof of his knowledge of science and engineering principles applied to his art, which enables him to control a biological process with such exactness. In all baked products made from fermented doughs there is involved the control of yeast fermentation and enzyme activity. In products made from unfermented doughs, such as cakes and cookies, the laws of physics play a very important part, as well as the principles of chemistry.

The industry itself is not fully aware of the importance of the chemist. This is the age of science, and in spite of the depression we are far better off than at any other time due to the products of scientific discovery. That industry which has not benefitted either directly or indirectly by the work of the chemist is truly far behind in the march of progress. Baking is both a science and an art. Its fundamental principles are based on physics and chemistry, but the skill and craftsmanship of the baker will never be supplanted by pure application of science.

Most of the experimental work to date has been done by

chemists in fields allied to baking. The yeast companies, the flour millers, the shortening manufacturers, the dairy firms, and others have spent large sums of money to employ the greatest chemists to work on bakery problems. The baking industry has profited by this work because the investigators, in order to solve their own problems, have been forced to study the principles of baking. Such outstanding authorities as Dr. C. H. Bailey, of the University of Minnesota, undoubtedly the most highly regarded cereal chemist in America, devotes much of his time to research work involving bakery problems. Dr. J. R. Katz, the eminent Dutch scientist, is probably the world authority on the staling of bread, a problem which is as yet but only slightly understood.

At the most recent National convention of bakers one of the speakers was H. S. Mitchell, chemist of Swift & Co., who attempted to point out to bakers the great possibilities which lie ahead in research work in the bakery field. To illustrate the common problems of bakers which are as yet unsolved chemically, Mitchell asked bakers a few simple questions. What are the effects of the various ingredients on the emulsion in the cake batter? What methods of mixing will lead to the most satisfactory results? How may bread staling be retarded? What is the cause of characteristic bakery cake flavor? What happens to fat used in the deep fat frying kettle? What causes holes in bread and how can they be avoided? These questions are but a few which will be answered ultimately by research chemists.

It is not my intention to present to you the fundamentals of the application of chemistry and science to baking, but rather to impress you with the fact that baking involves much basic science, and since baked products are so close to the people, it is well that they have at least a superficial knowledge of the materials and processes involved, and an appreciation of the problems of the baker, who usually is not technically trained, but who, especially in recent years has made great efforts to obtain such technical knowledge as will enable him to improve his skill as a craftsman.

Another very important reason why students should be better acquainted with chemical procedure is to obtain an appreciation of the value of chemical work. In the baking industry, as an example, where bakers for the most part are without technical training, many bakers send samples of ingredients to chemical laboratories for testing. The laboratories report their findings in

a very few figures, and many times in technical terms which are quite unintelligible to the baker. The report is usually accompanied by a bill. The baker sees a sizeable bill and but a few figures and feels that he has been overcharged. Fully realizing that much of the fault in this example lies with the laboratory which makes such an unintelligible report, I still feel that if students were given an appreciation of what is required to obtain the few figures which form the basis of the report, they would be more willing to cooperate with chemists, and to understand why years of training are necessary to adequately equip a man for doing laboratory work. The interpretation of chemical analyses is far more important to the industrialist than the analyses themselves. It is my opinion that students in chemistry should be given more of the ability to read and interpret chemical analyses than to make them. The most valuable employee is the one who can take a laboratory report and explain to the bakery superintendent just what should be done to turn out uniform bread of good quality with that particular sample of flour or other ingredient. Baking has not yet reached that stage of scientific control where such conditions are always possible. The most expert bakers and baking chemists still disagree on many basic practices in the plant.

There is no doubt in my mind, having been originally trained as a chemical engineer, that a sound training in chemistry, mathematics, and the general sciences is of far more practical value in making one's way through life, than are so-called courses in liberal arts. There are so many ways one can use the fundamental scientific knowledge in his every day work no matter what that may be, that it is my opinion that in this day no general course of study should permit the student to escape some training along this line.

There is a very definite desire on the part of hundreds of bakers to learn more about the application of chemistry and physics to baking, but they can not take complete courses in the sciences because much of the material is irrelevant, and the time occupied is too great. So strong has that desire become that an organization known as the American Society of Bakery Engineers was founded in 1924. This group, of which I happen to be secretary, is conducting a unique experiment in self education. It is composed of bakery superintendents and workers, bakery chemists, chemists engaged in allied lines, bakery equipment manufacturers, and others interested in the production of qual-

ity baked goods. It meets annually in Chicago and conducts meetings which are practically classes in baking, the teacher or speaker, being some member who is particularly proficient in some phase of baking. It conducts practical research work in the bakery plants of its members by testing out theories on a large scale and then collecting the data and drawing conclusions. It conducts an information service for its members at all times. A question is asked and through the Service Bureau that question is sent to three members of the Society best qualified to answer that particular question. The inquirer receives back through the Service Bureau, three answers which may or may not agree depending on the experience of those who answered the question. None of the four members have knowledge of the other three involved in this process so there is no possibility of getting a biased answer. This service is conducted because as yet there is no source of authentic bakery information based on scientific study, except in a very limited amount.

My experience has been such that I am not unmindful of the problems which confront you as teachers. For seven years I taught experimental baking, the fundamentals of baking, and bakery arithmetic, in a baking school. My classes included students with no specific prerequisites. Imagine teaching elementary arithmetic and applied chemistry to a group of students among whom were graduates in chemistry from recognized universities and others who many years ago left school after having completed four or five grades, some with twenty-five or more years of bakery experience and others mere boys with no knowledge of baking. I realize the difficulties and problems which teachers have in interesting students in work of this kind. This experience is mentioned, however, to indicate that since your students are more nearly equal in background and experience your problem is much simpler.

For an industry the size of the baking industry there are comparatively few places in which bakers, and young people coming into the baking industry to seek their fortunes, can obtain the technical knowledge they want and need. There are a few baking schools, but the tuition is fairly high and the time spent away from work and the cost of living in a distant city make such training almost prohibitive to many prospective students. Here in Chicago, through a fine spirit of cooperation existing between your Vice Chairman, Mr. Davis, and the instructor in baking in his school, Mr. Roser, we have an excellent example of what can

be done in a public high school to make better craftsmen out of young men who are earning their living in the bakery. One other public school, to my knowledge, has a course in baking, the Frank Wiggins Trade School of Los Angeles, California. I know of no other place where free instruction in baking may be obtained. A few colleges and universities have put in courses in connection with the courses in chemistry and home economics but usually these courses and instructors are so far removed from practical baking that their work is not of the kind which is in greatest demand.

There is a real opportunity for classes in night schools and in vocational schools on the general subject of baking. In my capacity as editor of a bakery magazine, I am in a position to know how many people in the industry write to us frequently asking where they may obtain such instruction at little or no cost, and in their communities. My thought is that schools, particularly public schools, can greatly enhance the respect they can obtain from business men in the community if they teach material which is directly applicable to industry. Too frequently chemistry teachers lose contact with industry and begin to teach abstract chemistry which is of very little value, except as training the student who does not intend to make chemistry his life work. Bakers everywhere are eager to cooperate in educational work. They would welcome an opportunity to study chemistry if they could see in it a connection between theory and practice. Can't you see how dull and uninteresting it would be to a baker with no chemical training to study the temperature volume relationship of gases, or the caramelization of sugars by heat? But if you explained that when he placed his pan full of dough into the oven the sudden increase in temperature caused the gas in the dough to expand and the gluten to stretch until the temperature was reached at which the rubbery gluten lost its elasticity and the loaf took on color, he would become thrilled at once with his new practical knowledge. That is the difference between pure and applied chemistry and in many cases between successful and unsuccessful teaching.

For two reasons I eagerly accepted the invitation to prepare this paper. First, I hoped to bring to you a better appreciation of the baking industry as a chemical industry, and one which affords opportunities for your students if they apply themselves to it. Second and most important, I hope that at least one of my readers will leave with the thought that he will contact a pro-

gressive baker in his community, become better acquainted with the educational needs of bakers with respect to chemistry physics, and mathematics, and perhaps attempt to develop a very simple, elementary course in applied science which will be available to those who care to avail themselves of the opportunity.

FIELD WORK IN AN AGRICULTURAL AREA*

By Clinton Rich
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Since I have been asked to tell you about the methods used in the field work on which I based my dissertation for a Masters Degree at the University of Chicago, I shall confine myself to telling what I did rather than making generalizations about what others should do except for some suggestions based upon the success or failure of certain phases of my work.

Before deciding to do any field work the student must ask himself this question: "Have I sufficient training under supervision to have a chance of success individually?" It would seem that at least one field course under a competent instructor is essential before any individual work be started. Before beginning my project I had had the good fortune to have had a field course from Professor Mark Jefferson of the Michigan State Normal College and another from Doctor Robert Platt of the University of Chicago. It is the methods taught by the latter, adapted to my particular problems which carried me through my work. In organizing my findings the advice and criticism of Doctor Jones of the University of Chicago was invaluable.

Selecting an Area. After deciding to do field work the first major problem is the selection of an area in which to work. The interests of the student will largely guide his decision. For one doing his first piece of individual field research, care must be taken not to be too ambitious for at best the work involved will be several times as great as estimated.

The next question should be: "Is the area simple enough so that it may be interpreted without going too far afield?" The area which I chose was distinctly an agricultural community without a single urban settlement of any kind. It was also very definitely bounded, for it was a drainage district in the flood

* Read before the Geography Section of the Central Association of Science and Mathematics Teachers, Dec. 1, 1933.

plain of the Mississippi River and of course had definitely established boundaries. The establishing of boundaries may become a difficult problem for the investigator if an indefinite area is chosen.

"Is the area small enough so that it can be thoroughly investigated in a reasonable length of time?" is a question which can not be overlooked. The area which I chose was approximately ten miles long and four miles wide. It contains about twenty-three thousand acres. Since it must be mapped in detail by pacing, it is about as large an area as one individual can hope to investigate thoroughly in the Saturdays and vacations of one year.

Obviously any area worked upon must be available to the worker. Bay Island was located only seventeen miles west of Aledo, Illinois, the city in which I was teaching at the time.

Any large piece of field work should try to answer a useful question which has not been previously handled if it is to be of any value other than practice for the worker. The question which I set myself in the study of Bay Island Drainage District was, "What adjustments have the farmers of this district made to the special conditions under which they must work?"

Mapping. The most important part of the field work proper was the mapping. The method used was the fractional complex system which is taught by Dr. Platt in his northern field classes.

Using the fractional complex system a fractional form is used to show both the natural conditions and the cultural uses of the land. The natural conditions are shown by code numbers in the denominator and the uses of the land and other cultural features are shown by the code numbers of the numerator.

Since the quality of the drainage was the dominating natural condition of Bay Island Drainage District the first number of the denominator showed drainage. Number one represented well drained land. Two represented land which was adequately drained most of the time but suffered during wet seasons. Three represented land that was well enough drained to produce crops only in unusually dry seasons; four, land that is too wet to be used for crops but may be pastured; five, swamp land which may be crossed by man or animals; six, swamp land so wet as to be a definite barrier to both man and beast; seven, open water.

The second number showed the type of soil. Number one showed clay; two, loam; three, sand; four, muck.

In the numerators the first figure showed the primary use of

land. Number one represented cropped land; two, pasture; three, woodland; zero, waste land. If the land was cropped number one in the second position represented corn; two, oats; three, wheat; four, leguminous hay; five, mixed hay; zero, not planted in 1929. The minor crops were also shown by numbers or letters. Pasture was designated in the second digit by one for natural blue grass, two for wooded pasture, three for marsh grass pasture, four for hog pasture, five for rotation pasture. If the first figure showed woodland, number one in the second digit represented heavy woods and two light second growth timber. The third digit in the numerator showed the quality of the crop and was used only on cropped land and pasture. Number one showed a good crop, two a fair crop, three a poor crop, and four a failure.

In mapping, the boundary of each change in complex whether in cultural or natural condition was actually drawn upon the map in the field. The condition on each side of this boundary could then be shown in a very small space by the fractional code. When field mapping was finished and a working map made each complex could be easily and accurately read.

Before actual mapping began every available detailed map of the area was studied and either permanently obtained or copied for use in checking later work. These proved a great aid but most of them were so idealized as to be of limited value or so old as to be obsolete.

The materials used in mapping were a plane table, rule, and pencil for mapping by pacing, by which over three-fourths of the area was mapped. A transit was used to check by triangulation the area previously mapped by pacing and also the inaccessible areas of marsh and water. The transit is not recommended as a standard equipment for geographical field work but since a high bluff bounded one side of the district and overlooked considerable of the inaccessible marsh land I found it most convenient to map this part by triangulation. Several square miles could be accurately surveyed in a single day by this method without the expense of hiring chain men or any other help.

Pictures as Supplementary Material. Pictures form a valuable addition to any field research. They can prove debatable points, illustrate examples given in the text, and make the whole more concrete and interesting to the reader. Care must be taken, however, that pictures have geographical value. Unless the

picture shows an actual relationship between a human activity and natural environment or is of such nature that a relationship can be developed from it, the value is slight indeed. The taking and using of pictures promiscuously whether they have any relation to the text of the completed manuscript or not, merely to have the work "copiously illustrated" is an inexcusable waste of time and money. However, in most cases many more pictures must be taken than are used for they must be taken before the manuscript is written and many that we expect to use at the time will not fit in with later plans and we often obtain better pictures of the same type later.

Use of the Questionnaire. Many research field workers have good success with questionnaires but I never have. Perhaps I am not enough of a diplomat to get intelligent answers to the questions or perhaps I expect too much. Most of the objections to answering a questionnaire were based upon fear or ignorance. One man insisted that I was asking him questions so that Wall Street bankers could take his farm away from him more easily. The questionnaire is of but little value unless a large proportion of the farmers will answer, which they did not do for me.

Informal interviews were far more successful. A farmer loves to talk and especially to tell how to run things. He will gladly tell many times as much while just "gassing," as he calls it, as he will in a formal interview or on a questionnaire. Some are flattered and will speak more freely if notes are taken while they talk, but others will shut up instantly when they see pencil and paper. I became well acquainted with several farmers in the district and obtained a great deal of valuable information from them. Considerable diplomacy is necessary in making these contacts, but in most cases I was able to find out all I needed to know.

The Use of Books and Other Data Collected by Others. If there has been work done on the same area previously by all means use it. Bay Island had never had a geographical study made, but some other work had been done. A land survey was in the county court house. It showed only roads and property line but was useful nevertheless. A complete soil survey has been made of Mercer county by the Illinois State Department of Geology. This was very useful but was naturally too generalized to be depended upon for so small an area. The U. S. war department publishes a report every year on all drainage districts along the Mississippi, which strangely enough, are under its

jurisdiction. This gives an analysis of costs, water pumped and comparative efficiency of each pumping plant. This work was proved of great value. Last but not least remember that the U. S. Weather Bureau has a station in or near any area chosen in this country.

Making the Final Maps. After the field map was finished the big job was to make useful maps from it of suitable size to be used in the dissertation. A tracing of all pieces of the field map was made on tracing cloth and sent to a reproducing company to be reduced and printed. These reproductions could then be used as a base map upon which different relations between human activities and natural conditions could be portrayed. The maps made for this area were: drainage, soils, land utilization, ownership and operating units, distribution of corn by drainage, distribution of wheat by drainage, distribution of hay and pasture by drainage, distribution of marsh and woodland by drainage. The same colors and symbols were used to portray the same things on the various maps as far as possible.

Interpretation of Data. Having collected the data and prepared the maps, the work is well begun. The next step is interpretation of data. Tables showing production, acreage, and distribution must be prepared and related to each other and to such data as are not in tables. For instance in Bay Island Drainage District we find that corn covers 59.8% of the cropped land of the district, 8,171 acres of corn, 60% of the corn is shelled and sold as grain, 53% of the corn is on well drained land, and a host of other facts which taken singly are meaningless but must be related to each other in order to answer the question which we set out to study.

Conclusions. One thing remains, the summing up of conclusions about the area. On this one point instead of telling how I did it I shall merely tell the conclusion which I reached. Bay Island Drainage District is one of the most successful reclamation projects on the upper Mississippi, but—every acre must pay an average of $3.50 drainage tax per year. The crops drown or are damaged often enough so that the yield is less than on the upland nearby which produces the same crops and whose farmers are clamouring for aid. Judge for yourself if it is successful. Naturally conditions in the district are much different than on the adjacent upland but the farming methods are nearly the same. The farmers simply have not made all the necessary adjustments to the natural conditions.

DIMENSIONALITY*

By Ernest P. Lane
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We have been told that the subject of an address should be brief and simple, and that it should be so worded as to arouse curiosity and interest in the minds of prospective auditors. Our subject of one word "Dimensionality" is certainly short enough to win the approval of even the most fastidious in this respect. Whether or not it aroused curiosity in your minds as to what the content of the address would be I cannot say. However, I can that while meditating on the subject recently my own curiosity was aroused to speculate upon what ought to be said on the subject this afternoon. Please do not misunderstand me! The difficulty is not in finding something to say appropriate to the subject, but rather it is to select out of all that might be said the right thing to say in twenty minutes to interest a varied audience.

First of all, the subject can be qualified. If your speaker were a physicist, and you were attending a meeting of the American Physical Society, and would expect to hear something about dimensionality probably quite different from what you actually do expect to hear, since your speaker is a geometer and you are attending a meeting of mathematicians. In fact, our subject means *dimensionality in geometry*, or perhaps *dimensionality of space.*

This subject can be treated in various ways. For example, a very interesting lecture could be prepared on the subject "Changing Ideas of Space." The proper mode of approach to this phase of the general subject would be to consider it chronologically. One could begin with the earliest known origins of geometry. The Egyptian geometer of 1000 B.C. was a very practical man, a surveyor of land, or a builder. He probably did not concern himself very much about the abstract idea of space. The Greek geometer of the time of Euclid, 300 B.C., had a rather definite idea of space as a "locus in which," that is, a locus which contained his figures and in which they could be moved about and compared. This conception of space was the dominant one for more than two thousand years. In the eighteenth century

* Read at the Mathematics Section of the Central Association of Science and Mathematics Teachers, Dec. 1, 1933.

A.D. it was suggested that our ordinary space of three dimensions with time as a fourth dimension leads to the consideration of a four-dimensional world, but this idea did not grip the public imagination till early in the twentieth century. In the meantime, the development of projective geometry by Poncelet in 1822 and by his followers brought about the first general modification of the definition of space. *Projective space* is quite different from euclidean space. In projective space, for instance, two coplanar lines always intersect, but this is not true in euclidean space. Cayley (1821–1895) took the step from a four-dimensional space to an *n*-dimensional space. Later, spaces of denumerably infinitely many dimensions were considered, and still later, about 1900, spaces of continuously infinitely many dimensions. In the meantime Riemann in 1854 introduced the preliminary ideas of an entirely different conception of space, which was afterward used by Einstein in his general theory of relativity. According to Riemann and his followers space is not a "locus in which." For them there is no such thing as an empty space. The space itself is the object of interest. Perhaps the best description of this point of view can be found in the Cambridge Tract No. 29, *The Foundations of Differential Geometry*, by Veblen and Whitehead. For them geometry is the theory of a space, and a space is a set of elements with a structure, that is, with a set of properties, the word *properties* being used in a very general sense. This would be one way to treat the subject of dimensionality.

Another way would place the emphasis on an entirely different aspect of the subject. An interesting lecture could be prepared on the subject "Alchemy, Astrology, Numerology, and Dimensionality." The proper avenue of approach to this phase of the general subject would lie in a consideration of the contrast between the legitimate sciences of chemistry, astronomy, number theory, and geometry and their bastard offspring. If there should be any one of you who believes that there is some secret mystical connection between the stars and his destiny, or if there is any one who thinks that positive integers have miraculous powers over his fate, far be it from me to attempt to disillusion him! However, true science is often misunderstood by the ignorant and credulous, and the avaricious cupidity of dishonest scoundrels has been known to turn into a sucker racket a spurious imitation of scientific procedure.

Let us now turn to the specific subject of dimensionality.

Popular interest in this subject, and particularly in *the fourth dimension*, was stimulated by the publication of Einstein's first theory of relativity in 1905. By 1909 popular interest was so great that an anonymous donor gave to the *Scientific American* the sum of five hundred dollars to be used as a prize for the best popular explanation of *the fourth dimension*, in not more than 2500 words. There were 245 essays submitted by authors living in almost every civilized country in the world. The prize was awarded to Lieut.-Col. Fitch, U. S. A. The winning paper was published in the *Scientific American* in July 1909, and in 1910 a number of the best essays were published in a book entitled "The Fourth Dimension Simply Explained," which is still interesting reading.

It was natural that some of the results of scientific inquiry were immediately seized upon by unscientific people who proceeded to exploit them for their own ends. Among these were writers of romances, prestidigitators, and a certain class of spiritualists. Authors found that they had at their disposal a new method for the appearance or disappearance of the hero or villain. A villain who understood the fourth dimension could suddenly disappear from a tightly closed room and appear at will somewhere else. The sleight-of-hand performers found and adopted a new set of catchwords and phrases with which to mystify their audiences. To some spiritualists the fourth dimension became the abode of spirits, from which they might be induced to return to our world by the right persons for a consideration. There were those who even went so far as to quote Scripture to prove the existence of the fourth dimension, and believed, for example, that the Apostle Paul was referred to the fourth dimension when he wrote to the Ephesians:

"That ye, being rooted and grounded in love, may be able to comprehend with all saints what is the breadth, and length, and depth, and height."

One reason for the widespread popular interest in dimensionality is that there are certain analogies which suggest very striking properties of space of higher dimensions than ours. Some of these analogies may be worth considering briefly. They consist of easy generalizations. For example, it is often said that a moving point generates a curve; a moving curve generates a surface; a moving surface generates a solid; then what does a moving solid generate? Again, if a is the length of a straight line segment, then a^2 is the area of the square with this segment

as a side; a^3 is the volume of the cube with this segment as edge; but what are a^4, a^5, $\cdots$?

There are several analogies of a primarily analytic nature. For example, on a straight line S_1, one linear equation,

$$ax+b=0,$$

represents a point S_0. In a plane S_2, one linear equation,

$$ax+by+c=0,$$

represents a straight line S_1. In ordinary space S_3 one linear equation,

$$ax+by+cz+d=0,$$

represents a plane S_2. So in space of four dimensions S_4 one linear equation,

$$ax+by+cz+dw+e=0,$$

represents an ordinary space S_3, and so on. In a space of n dimensions S_n, one linear equation is said to represent a hyperplane S_{n-1}.

Again, on a straight line S_1 one quadratic equation of the form

$$x^2=a^2$$

represents two points. In a plane S_2 one quadratic equation of the form

$$x^2+y^2=a^2$$

represents a circle. In ordinary space S_3 one quadratic equation of the form

$$x^2+y^2+z^2=a^2$$

represents a sphere. So in space of four dimensions S_4 one quadratic equation of the form

$$x^2+y^2+z^2+w^2=a^2$$

represents a hypersphere. In a space of n dimensions S_n, one quadratic equation of the form of a sum of squares equal to a constant represents a hypersphere.

Finally, the distance between two points P_1, P_2 in space S_n is given by the formula

$$[(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2+\cdots+(l_2-l_1)^2]^{1/2},$$

the number of squares being the dimensionality n of the space S_n $(n = 1, 2, 3, \cdots)$.

Some of the most striking properties of hyperspace can be easily described. For example, there is what may be called the property of escape. On a straight line two points will confine an individual. But he can escape from confinement if the line is in a plane and he is permitted to step off the line onto the plane and go around one of the imprisoning points. A linear dweller, however, could not see where the individual went before he mysteriously appeared on the other side of the point. Similarly, in the plane a circle will confine an individual. But he can escape from confinement if the plane is in ordinary space and he is permitted to step off the plane into the ordinary space and go over the imprisoning circle. A flat dweller, however, could not see where the individual went before he mysteriously appeared on the other side of the boundary of the circle. Finally, in ordinary space a sphere will confine a person. But if he is allowed to pass into four-dimensional space he can escape from the sphere without going through its surface and to the amazement of us three-space dwellers appear again outside the sphere.

Another interesting property may be called the property of inversion. A rubber band in three-dimensional space can be turned inside out without tearing it, simply by making the particles of rubber roll past each other suitably. In the same way a hollow rubber sphere, such as a tennis ball, can be turned inside out in a space of four dimensions, without tearing the ball, by simply making the constituent particles of rubber roll past each other suitably.

Finally, there is the property of superposition. In a purely plane geometry it is possible to have two triangles whose sides and angles are all respectively equal, but which are nevertheless not superposable. Of course, if one triangle is taken out of the plane and turned over, then the triangles are superposable. Similarly in space of three dimensions it is possible to have two figures whose parts are all respectively equal, but which are nevertheless not superposable. For instance, your right-hand glove does not fit your left hand. But if you take your right-hand glove out into four-dimensional space and turn it over, then it will fit your left hand.

Having considered dimensionality from the non-professional point of view, let us now become more critical, and formulate a definition of dimensionality. For this purpose let us review the

definition of analytic geometry according to Klein. According to this famous German geometer, one would first select the *space* of the geometry, for example, the *plane* if one wished to study plane geometry. Then one would select the *element* of the space. It is most common to select the *point* as the element, but students of projective geometry use also the straight line as element, and the circle could be used, or the parabola, or the general conic. The next step is to construct *figures* out of the elements. The next is to select a *group of transformations* to which the figures are to be subjected. For example, in metric geometry the group of motions is used. A figure can be moved from one place to another without changing any of its metric properties. The geometry then consists of those properties of the figures which are unchanged by the transformations of the figures.

If it is desired to study analytic geometry it is necessary to establish a *coordinate system*. This is any one-to-one correspondence between the elements of the space and ordered sets of numbers, for instance, a one-to-one correspondence between points in a plane and pairs of numbers (x, y). The numbers are called coordinates of the element, and will be supposed to be real. The definition of dimensionality can now be stated from the analytic point of view.

The dimensionality of a geometry is the smallest number of coordinates necessary and sufficient to locate an element in the space of the geometry.

Before looking at some illustrations, it should be observed that the dimensionality of a geometry depends on two things, namely, the *space* and the *element* of the geometry. This should not be surprising. Even such an elementary thing as the distance between two points depends on two things, namely, the points themselves and the unit of distance used. If one investigator reports that the distance from the Congress Hotel to Eckhart Hall is seven and another fifty-seven, do not be alarmed. The one investigator is using the mile as the unit, and the other the city block.

According to our definition of dimensionality *it is possible to construct a geometry of any number of dimensions in any given space by choosing the element suitably*. For example, in the plane we have a two-dimensional geometry if the point is the element, and also if the straight line is the element. But if the circle is the element, the dimensionality of the geometry is three, be-

cause it takes three coordinates to locate a circle in a plane, say the two coordinates of the center and the length of the radius. Similarly the dimensionality of a plane geometry with the parabola as element is four, and with the general conic as element is five. In ordinary space the dimensionality of the geometry with the point as element is three, and the same if the plane is the element. But if the straight line is the element the dimensionality is four, and the same for the sphere. If the circle is the element the dimensionality is six; for the conic it is eight; and for the quadric it is nine. So a concrete realization of a geometry of any number of dimensions can be established in our familiar space of experience.

In conclusion, you may be tempted to ask what is the use of all this. To be sure, you can not take these ideas to the store and exchange them for dry goods. But they are useful in our business in spite of that. If the argument that the study of hyperspace is interesting to a professional geometer is not regarded as convincing, it may be worth mentioning that the study of hyperspace enables us to understand our own space better. But perhaps you are more interested in the utility of these ideas for the teacher of high-school geometry. It goes without saying that one of the aims of all of us is to interest our students in the subject we are teaching. To a great extent our success is measured by the amount of interest and enthusiasm we kindle in our class rooms. To stimulate interest in geometry on the part of a certain category of our clientele requires the presentation to them of something more vital than the dust and dry bones of logic as handed down to us by Euclid. My personal experience spread over twenty-four years of teaching has convinced me that there is no way in which it is easier to arouse discussion and resurrect a dead geometry class than to inject some of the preliminary ideas of dimensionality.

EXAMINERS FOR BOY SCOUT CHEMISTRY MERIT BADGE

If you have not already done so, please send your name, address and the name of your Council to

R. A. BAKER,
Chairman, Division of Chem. Education,
Lexington Ave.,
New York City.

THE SCIENTIFIC ATTITUDE AND SKILL IN THINKING

By Elliot R. Downing
University of Chicago, Chicago, Ill.

The article in the December number of School Science and Mathematics (Vol. XXXIII, 964–68) on "What is a Scientific Attitude?" demands comment for it tends to confuse two quite distinct things—the scientific attitude of mind and skill in scientific thinking. The author says that a satisfactory definition of scientific attitude can not be secured. Yet before discussion can proceed the term must be defined otherwise confusion will be still more confounded.

The term 'scientific attitude' as here used means that attitude of mind which impels one to attempt the solution of problems which he faces by scientific thinking or good reflective thinking —rather than by the trial and error method. The scientific attitude is more than an intellectual assent to the proposition that the scientific method of thought is a desirable method of procedure. It is an impelling attitude and is therefore at least partially saturated with emotional elements. The scientific attitude is a fervid conviction that problems within the range of science can only be successfully solved by the scientific method of thinking. The fervor is somewhat akin in its intensity to that of a religious belief. One is more or less possessed by the ardor of his conviction.

One judges, then, how completely the scientific attitude dominates a man by noting in how far he has recourse to the scientific method in solving his problems. And that is the sole criterion for judgment. If he tries to solve some specific problem by reflective thinking, he manifests in this instance the scientific attitude. If he habitually tries so to do when problems arise the scientific attitude is well established.

But he may be quite unskilled in scientific thinking and so in nine cases out of ten reach wrong solutions. The success of his efforts depends on his skill in thinking, not on his scientific attitude.

One becomes skilled in scientific thinking as he acquires any other skill, by *first* learning what are the elements involved in it and what the safeguards that must be observed in their proper use and *second* by practice. The list of "characteristics exhibited

by persons having a scientific attitude" on page 965 of the article certainly includes several of these elements and safeguards. Observation, judging on the basis of facts (Nos. 7 and 4) are elements or steps in the process of scientific thinking, while a realization that judgments must be impartial, suspended until the evidence is adequate, and changed when new evidence demands (Nos. 11, 12, 6) are safeguards that must always be kept in mind when judgments are being formed.

To try to determine what are the elements and safeguards of scientific thinking by the questionnaire method is totally unscientific. Facts should be collected, not opinions. The writings of the great scientists, their records of procedure in the discovery and application of the laws of science, are the source materials from which this list must be determined by careful observation and analysis.

One would not submit a sample of an unknown to chemists to get their offhand opinions on its composition. Rather would it be sent to a reliable chemist for analysis. He could tell for certain what elements it contains. Here again it is a matter of fact, not of opinion. One might wish to have the facts confirmed by a second or even a third analysis. If the chemists selected are experts their analyses will agree.

So experts in the field of scientific thinking who are familiar with the reports of investigations of great scientists should and really do agree pretty completely on the elements of the process and the safeguards that must be observed in their use. Such confusion as exists arises largely from differences in terminology not from disagreement on the facts.

Summary. This article tries *first* to define scientific attitude as here used; *second*, to distinguish sharply between scientific attitude and skill in scientific thinking; *third*, to make clear that the list of elements and safeguards of scientific thinking must be determined on objective evidence—on the basis of fact —not on opinions.

SCIENCE QUESTIONS

Conducted by Franklin T. Jones, 10109 Wilbur Avenue, Cleveland, Ohio.

Readers are invited to co-operate by proposing questions for discussion or problems for solution.

Examination papers, tests, and interesting scientific happenings are very much desired. Please enclose material in an envelope and mail to Franklin T Jones, 10109 Wilbur Avenue, Cleveland, Ohio. Thanks.

HUDSON, OHIO—MARCH 17, 1934

Dr. Joel Hayden, Headmaster of Western Reserve Academy invites Science and Mathematics teachers "hereabouts" to meet in Hudson.

A.M.—Supt. Charles H. Lake of Cleveland will be the speaker.

P.M.—Nature Guide School Annual Outing in the sugar camp.

Evening—Dinner and entertainment in Cutler Hall.

YOU ARE INVITED TO COME!

HOW COLD IS ICE?

645. *Proposed by A. M. Shofner, Shelbyville #6, Tenn.*

Freeze a Fahrenheit thermometer in a block of 100 lbs. ice. Though our temperature would go to 110° in summer, it would still stand at 32°. Then in case we throw the block of ice out into the yard in winter, and our mercury goes to 10° below zero—what will the thermometer then show in the ice?

THE FLY AND THE BICYCLES

646. *A Clipping from B. Felix John, Catholic H. S. Pittsburgh, Pa.*

Once upon a time there was a stretch of road twenty miles long. At each end of this road was a bicyclist, and at exactly the same instant they started riding toward one another at a constant speed of ten miles an hour, continuing until their front wheels touched. At the instant of their starting, a fly, which was perched on the front wheel of one of the bicycles, started to fly toward the other at a speed of fifteen miles an hour. He flew until he touched the other front wheel and instantly started back, always at the same speed, till he touched the front wheel of the first, and so on, flying back and forth between the wheels until he was crushed as they met. Each journey naturally was shorter than the one before.

Now all you have to do is to figure out—exactly how far did the fly fly?

A BYRD OF A QUESTION!

647. Here is a clipping from a recent issue of the *Chicago Herald and Examiner* telling what Admiral Byrd found when he reached "Little America." What type of lighting system is in use?

"Somebody cranked the phonograph, and at the first strain of music, Byrd burst into laughter. The tune was 'The Bells of St. Mary's.' Quin Blackburn used to play it twenty times a day, and he still likes it!

"Petersen idly flipped a switch and the lights of the building went on. He pressed the telephone connection in the Administration Building. Dr. Poulter picked up another phone. After four years, everything worked."

AMMONIA BURNS!!

648. *Proposed by Kenneth L. Goding, Attleboro H. S., Attleboro, Mass.*

During our last laboratory period we made ammonia by the usual method of heating $Ca(OH)_2$ and NH_4Cl. On testing the gas with a lighted splint the gas lighted. I had used the same mixture to make ammonia several periods before and the gas did not ignite then. I have done this experiment many times before and have not seen the ammonia burn. Chemistry texts say that ammonia burns in an atmosphere of oxygen but where did the oxygen come from? Does ammonia burn?

RIPLEY'S PROBLEMS AROUSE INTEREST

632. A quart is $\frac{1}{4}$ of a gallon; a quart is $\frac{1}{8}$ of a peck; and a gallon is not $\frac{1}{2}$ of a peck. Why?

Solution by Charles W. Trigg, Cumnock College, Los Angeles, Calif.

Briefly, because all quarts are not equal.

The apparent paradox is caused by the ambiguity of the word "quart," and the consequent unwarranted assumption that all quarts are equal. Thus, there is an implicit substitution of "every" for "a", prior to the application of the axiom of equality.

As a matter of definition, a liquid quart (U. S.) is equal to $\frac{1}{4}$ of a gallon or 57.75 cu. in., and a dry quart is equal to $\frac{1}{8}$ of a peck or 67.20 cu. in. Whence a gallon equals 0.43 peck rather than $\frac{1}{2}$ peck, since a definite common unit of measure, the cubic inch, is employed for the comparison.

This curiosity cannot be ascribed solely to the difference between dry and liquid measures, for the magnitudes of the liquid measures indicated by the term "gallon" differ widely. By definition, the Imperial gallon (Breat Britain) equals 277.42 cu. in., the United States or wine gallon is 231 cu. in., the old ale and beer gallon is 282 cu. in., the gallon of Argentina occupies 233.31 cu. in., and the Peruvian gallon holds 205.59 cu. in.

The existence of such lack of standardization throughout our various systems of measurement permits the creation of a large list of similar Ripleyesque posers. Another one of ancient vintage is "A pound of feathers is heavier than a pound of gold." This is explained by the fact that feathers are usually weighed by the pound avoirdupois (7000 grains or 16 ounces), whereas gold is weighed by the pound troy (5760 grains or 12 ounces). Here the grain is the common unit. Evidently, it follows that an ounce of feathers is lighter than an ounce of gold.

Wherever ambiguity exists such situations arise. "A dog is a mammal, a dog is a piece of machinery, but a mammal is not a machine," is more patent than the aforesaid examples, but involves the same logical principles.

Ripley's problems are of pedagogical value in that they arouse that elusive factor, "interest." Their value may be increased materially by using them to emphasize the importance of exact definition, the fundamental nature of axioms, the utility of mathematical principles in general, and to solidify the foundations of abstract reasoning. Again, this type of ambiguity offers an excellent argument for the universal adoption of the metric system.

WANTED ANSWERS!

636. The ball drops through the Earth. Where does it come out? What happens on its journey?

Send in answers to help out the Physics Class at Norwich Free Academy.

GQRA

(Guild of Question Raisers and Answerers)

You become a member when you send in a question, or an examination paper, or a test, or an answer that is published.

NEW MEMBERS—MARCH, 1934

16. Alden M. Shofner, Shelbyville, Tenn.
17. B. Felix John, Pittsburgh, Pa.
18. G. W. Warner, Chicago, Ill.
19. Kenneth L. Goding, Attleboro, Mass.
20. Charles W. Trigg, Los Angeles, Calif.

Have your class answer or propose a question!! Others have done so.

ACTIVITIES OF THE COMMISSION ON THE TRAINING AND UTILIZATION OF ADVANCED STUDENTS IN MATHEMATICS

At the meeting of the Mathematics Association of America in Chicago in June, 1933, the Trustees of the Association requested the President to appoint a commission on "The Training and Utilization of Advanced Students in Mathematics." President Dresden has appointed the following commission: William Betz (Rochester High School, Rochester, N. Y.), W. L. Hart (Minnesota), J. O. Hassler (Oklahoma), E. R. Hedrick (California), E. V. Huntington (Harvard), M. H. Ingraham (Wisconsin), R. G. D. Richardson (Brown), H. E. Slaught (Chicago), E. B. Stouffer (Kansas), E. J. Moulton (Northwestern), Chairman.

At the recent Christmas meeting of the Association at Cambridge the Commission made a report of progress. In view of the present scarcity of openings in colleges and universities for teachers of mathematics, it appears that a considerable number of highly trained students in mathematics will need to look for positions in other fields. One of these fields is that of the junior colleges and secondary schools. With regard to this latter field the Commission recommended the publication of the following statements, and the transmission of them to graduate students in mathematics:

(1) Advanced students in mathematics who may become candidates for positions in secondary schools should (a) prepare to meet the legal requirements of the state in which they hope to teach, which requirements include a certain number of credits in Education and Practice Teaching; and (b) give intelligent consideration to their whole program of study in preparation for such work.

(2) The Commission is undertaking a study which may lead to the formulation of recommendations concerning the training on both the professional and subject matter sides of teachers of mathematics in the secondary and college fields.

The Commission feels that there is a very important service to mathematics to be rendered by thoroughly trained teachers of mathematics in the field of secondary education. Members of the Commission have expressed their conviction that problems facing such teachers merit the best efforts of competent young mathematicians.

BIG GATHERING OF SAPS IN THE CUYAHOGA COUNTRY

WRITTEN BY ONE OF 'EM

The land of the Cuyahoga is a U-shaped territory bounded by the Cuyahoga River. The river rises in Geauga County, its source being farther north than its mouth which is at Cleveland on Lake Erie. The base of the U is just north of Akron. In pre-glacial days the Cuyahoga reached the Atlantic via the Tuscarawas and the Gulf of Mexico. Today it swings north and finds the Atlantic by way of the St. Lawrence.

In the same way that the land of the Maumee in northwest Ohio grows all the Buckeye sugar beets the land of the Cuyahoga in northeast Ohio is the center of the state's maple sugar industry. That has a hidden relation to the fact that Geauga County imports more brown sugar than any other county in the country. All of this explains why Hudson the original home of Western Reserve University and the present location of Western Reserve Academy, in the heart of Ohio's sugar grove, has been selected for the Maple Sugar Frolic of the science and mathematics teachers of the Great Lakes Region.

For a month these teachers have been chatting about going down to Hudson on March Seventeenth. Since they are science teachers they should say "Up to Hudson" for it is in Summit County and besides doesn't the Cuyahoga flow down hill? After leaving the Lake Plain the route will be along SOM Center Road (the SOM standing for Solon, Orange Village, and Mayfield) which traverses the glaciated Allegheny Plateau. It is an intellectual shock to some to learn that as soon as they reach the top of Cedar Hill they have mounted the northern limit of the Alleghenies. All of these "sappy" things (How many can you find in these three paragraphs) may be used to impress the guest-pupils that the teachers are inviting to the "Sap" Party.

Here and there you will dip down into a valley or through a cut in the hill that will expose layers of horizontal rock. These rocks grew in what was formerly the Gulf of Mexico. If you are ahead of time, poke over the pebbles and rocks and you may find a marine shell—evidence enough of a Mediterranean Sea. Don't fail to take a peek at the Twinsburg Conglomerate with its white quartz pebbles. Stirring days when these pebbles were washed down from Appalachia to make Nelson, Boston, and Whipp's Ledges. These huge conglomerate blocks are 1000 ton crumbs that have been cut off from the original pudding by stream action. The fantastic forms are due to the tooth of time. You may even catch the Frost King pulling a few raisins out of the pudding. Motoring over the deposits of an ancient sea bottom ought to stir your imagination and the exercise will surely whet your appetite. Don't get so enraptured in reading the geological book that you fail to reach the Academy Chapel later than ten A.M. by the belfry clock.

In this century-old structure where pious young men were once prepared for the ministry there will be a gathering of teachers no less devout to a cause and actuated by equally deep feelings. Here they will listen to the message, if not exhortation, of Charles H. Lake, Cleveland's new Superintendent of Schools. Mr. Lake is not only a superintendent but a scientist and an author of a science textbook of some note.

Following the speeches of the morning and music furnished by the Academy boys the scientists, lank and some of them long feeling hungry, will hie unto the Academy dining hall for repast. Head-Master Doctor Joel Hayden will be master of ceremonies.

Then the gathering of the saps. Just as our forefathers did, we will off to the sugar orchard. "Hi" Simons will be on hand with his mules and stone

boat to collect the sap. It will appeal to your palate just the same way it did to your grandpap's and there will be much barter and trade amongst ye school mam's who wish to take specimens back to their school houses. With the zest of the March wind there is something appealing about harking back to the making of maple sugar which has been practiced since the days of the Erie Indians.

And then there is the annual sugar bush hike led by Cap'n Bill. He will point out the work of that most methodical woodpecker, the yellow breasted sap-sucker and the lesser creatures that come to Uncle Hi's sugar party to be partakers with his other guests. This Ancient Mariner has promised that the Harbinger-of-Spring will be responding to the great diurnal changes in temperature and will show the distinguishing features by which one may tell the sugar maple when it does not have a sap spout and bucket. And perhaps he will call up a black-capped chickadee to perform over head.

If you know the traditions of sugar bushing you will be sure to join the annual sugaring-off supper of the Nature Guide School which will be held at the village church. Aunt Hepsie promises all the fixin's with all the hot biscuits and maple syrup that you can eat. The husbands will have whittled out maple paddles for the stirring and making of maple candy. If you bring the children they too will store up a keen memory of the days when the whole household entered into the operations of the sugar camp. And who cannot afford to know the sugaring off as the pioneer knew it? (All this for 50 cents in Wampum.)

Then just as though we had not had a day of it Cap'n Bill will show colored lantern slides of the maple sugar industry from the time that the first flow of sap hits the galvanized pails to the big festival in the village church. It is then that we will have a complete understanding of one of the gifts of the Great Spirit, and why we make it an annual occasion of nature-worship.

Western Reserve Academy, Hudson, Summit County, Ohio
March the Seventeenth, Nineteen Thirty-four

PROBLEM DEPARTMENT

CONDUCTED BY G. H. JAMISON
State Teachers College, Kirksville, Mo.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the problems should be well done in India ink. Problems and solutions will be credited to their authors. Each solution, or proposed problem, sent to the Editor should have the author's name introducing the problem or solution *as on the following pages.*

The Editor of the department desires to serve its readers by making it interesting and helpful to them. Address suggestions and problems to G. H. Jamison, State Teachers College, Kirksville, Missouri.

SOLUTIONS AND PROBLEMS

Note. Persons sending in solutions and submitting problems for solutions should observe the following instructions

1. Drawings in India ink should be on a separate page from the solution.

2. Give the solution to the problem which you propose if you have one and also the source and any known references to it.

3. In general when several solutions are correct, the one submitted in the best form will be used.

LATE SOLUTIONS

1265. *Henry H. Shanholt, New York City.*

1310. *Proposed by W. E. Buker, Leetsdale, Pa.*

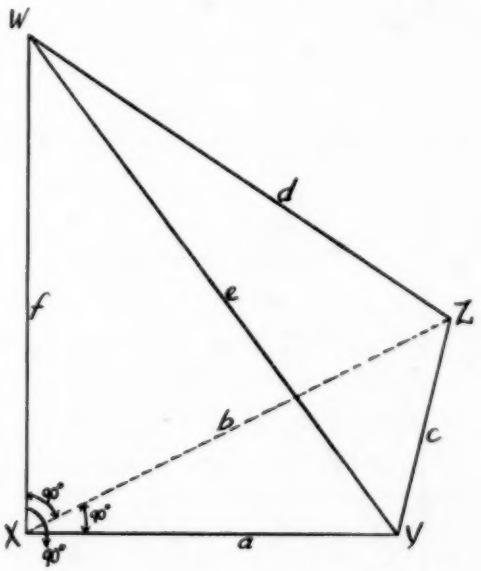
Find the length of the shortest and longest lines from the origin to the conic $ax^2+2hxy+by=C$. Find also the direction of these lines.

No solution has been offered.

1308. *Proposed by C. E. Sharp, Jr., St. Louis, Mo.*

In a right-angled tetrahedron, the square of the face opposite the right trihedral angle is equal to the sum of the squares of the three adjacent faces.

Solved by B. Felix John, Pittsburgh, Pa.



In the figure,

$$d=\sqrt{b^2+f^2},\quad e=\sqrt{a^2+f^2},\quad c=\sqrt{a^2+b^2},\quad \triangle XYZ=\frac{ab}{2},\quad \triangle XYW=\frac{af}{2},\quad \triangle XZW=\frac{bf}{2}.$$

Then,

$$(\triangle XYZ)^2+(\triangle XYW)^2+(\triangle XZW)^2=\frac{a^2b^2+a^2f^2+b^2f^2}{4}.$$

Now,

$$\triangle YWZ=\sqrt{\frac{c+d+e}{2}\cdot\frac{c+d-e}{2}\cdot\frac{c+e-d}{2}\cdot\frac{d+e-c}{2}}$$

$$= \tfrac{1}{4}\sqrt{[(c+d)^2-e^2][e^2-(c-d)^2]}$$
$$= \tfrac{1}{4}\sqrt{2e^2(c^2+d^2)-c^4+2c^2d^2-d^4-e^4}$$
$$= \tfrac{1}{4}\sqrt{(2a^2+2f^2)(a^2+2b^2+f^2)-(a^2+b^2)^2+2(a^2+b^2)(b^2+f^2)-(b^2+f^2)^2-(a^2+f^2)^2}$$
$$= \tfrac{1}{4}\sqrt{4a^2b^2+4a^2f^2+4b^2f^2} = \frac{\sqrt{a^2b^2+a^2f^2+b^2f^2}}{2}$$

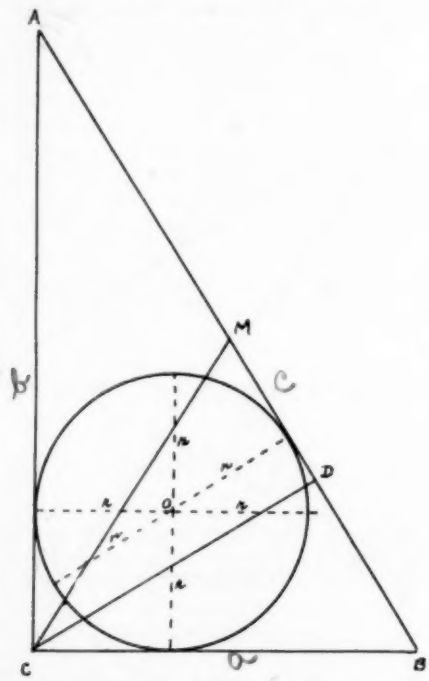
or $(\triangle YWZ)^2 = \dfrac{a^2b^2+af+bf^2}{4}$.

$\therefore (\triangle YWZ)^2 = (\triangle XYZ)^2+(\triangle XYW)^2+(\triangle XZW)^2$.

Also solved by Joseph L. Stearn, Washington, D.C.; W. E. Buker, Leetsdale, Pa.; John E. Bellards, St. Nazianz, Wisconsin; Charles W. Trigg, Los Angeles, California, and the proposer.

1309. *Proposed by C. W. James, Eldon, Ohio.*

Prove that the radius r of the circle inscribed in a right-angle triangle is less than one-half of either of the legs and also less than one-fourth of the hypotenuse.



First Solution by Harlan Reyburn, Petaluma, California

In the figure, let CM be the median to the hypotenuse, and CD the altitude.

$AC > 2r$, because the circle must be tangent to CB and can not go through A or be tangent to a line through $A \parallel BC$.

$$\therefore \quad \frac{AC}{2} > r \quad \text{or} \quad r < \tfrac{1}{2}AC.$$

Similarly, $r<\frac{1}{2}CB$.
Also $CD>2r$. But $CM>CD$. $\therefore$ $CM>2r$ and $r<\frac{1}{2}CM$.
Since CM is the median to AB, $CM=\frac{1}{2}AB$.

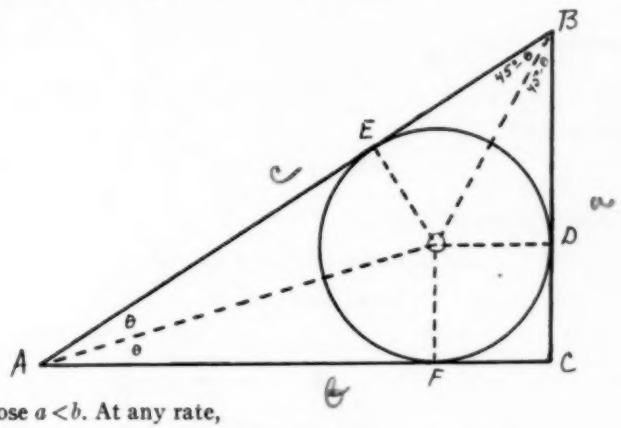
$$\therefore \quad r<\tfrac{1}{2}CM=\tfrac{1}{2}\left(\frac{AB}{2}\right)=\tfrac{1}{4}AB.$$

That is: $r<\frac{1}{4}AB$, the hypotenuse.

Second Solution by W. E. Buker, Leetsdale, Pa.

Let $AC=b$; $BC=a$; $AB=c$; $OD=OE=OF=r$; $OAE=\theta$; $OBE=45°-\theta$

(1) $\left.\begin{matrix}AF=AE=b-r\\ BD=BE=a-r\end{matrix}\right\}$ (since $ODCF$ is a square).



Suppose $a<b$. At any rate,

$$r=(a-r)\tan\theta \tag{1}$$

$$\text{But } \tan\theta<1 \text{ (since } \theta<45°). \tag{2}$$

From (1) and (2) $r<a-r$, and $r<\frac{1}{2}a$.

(2) $b-r=r\cot\theta$
$a-r=r\cot(45°-0)$

$$\therefore \quad a+b-2r=c=r[\cot\theta+\cot(45°-\theta)] \tag{3}$$

By means of a table of logarithms, or otherwise, we see that $\cot\theta+\cot(45°-\theta)>4$. See problem 1329.

From (3), $c>r\cdot 4$, and $r<c/4$.

Also proved by Herbert P. Leifer, Pittsburgh, Pa., and Charles W. Trigg, Los Angeles, Calif.

1301. *Proposed by B. C. Becker and L. E. Hebb, Woodriver, Illinois.*

A small sphere loses its balance at the apex of a larger sphere and rolls off. Neglecting friction, at what angle does its velocity become sufficient for the centrifugal force to carry it away from the larger sphere?

Solved by the Proposers

The centripetal force,

$$F_c=mg\cos\theta. \tag{1}$$

Acceleration is caused by the weight component:

$$F_a=mg\sin\theta.$$

Acceleration, $a=r\dfrac{d^2\theta}{dt^2}=\dfrac{Fa}{m}$ (θ in radians and r = radius of large sphere.)

$\frac{d^2\theta}{dt^2}=\frac{g}{r}\sin\theta$, and multiplying by $2\frac{d\theta}{dt}$:

$2\frac{d\theta}{dt}\cdot\frac{d^2\theta}{dt^2}=\frac{2g}{r}\sin\theta\frac{d\theta}{dt}$, which can be integrated:

$\left(\frac{d\theta}{dt}\right)^2=\frac{-2g}{r}\cos\theta+k$. Setting $\theta=0$, we obtain

$$k=\frac{2g}{r},\text{ then }\left(\frac{d\theta}{dt}\right)^2=\frac{2g}{r}(1-\cos\theta). \tag{2}$$

The centrifugal force,

$$CF=mr\left(\frac{d\theta}{dt}\right)^2. \tag{3}$$

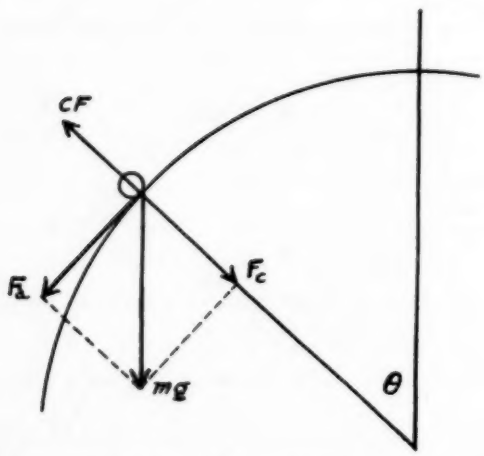
From (2) and (3):

$$CF=2mg\,(1-\cos\theta).$$

When $F_c=CF$ from (1) and (4):

$$mg\cos\theta=2mg\,(1-\cos\theta)\text{ from which}$$

$$\theta=\text{arc}\cos\tfrac{2}{3}=48^\circ\,15''.$$



Also solved by G. W. Grotts, Irving, Illinois; and Howard R. Harold, Tonkawa, Oklahoma.

1311. *Proposed by a Reader*

If S_p equals the sum of the pth powers of the first n integers, $S_{15}+7S_{11}+7S_{13}+S_9=N$, N, and integer.

Solved by W. E. Buker, Leetsdale, Pa.

By a suitable transformation of the Euler-Maclaurin formula for the approximate value for a definite integral, we get the following formula for the sum of the pth power of the first n positive integers:

$$1^p+2^p+3^p+\cdots+n^p=\frac{n^{p+1}}{p}+\frac{1}{2}n^p+\frac{B}{2!}pn^{p-1}-\frac{B_3}{4!}p(p-1)(p-2)n^{p-3}$$
$$+\frac{B_5}{6!}p(p-1)(p-2)(p-3)(p-4)n^{p-5}-\cdots. \tag{1}$$

(See, Whittaker & Robinson, Calculus of Observations, London, 1924, pp. 134–138.)

In (1), the B's refer to the Bernoullian numbers, as given in Pierce, "A Short Table of Integrals."

$$B_1=\frac{1}{6},\quad B_3=\frac{1}{30},\quad B_5=\frac{1}{42},\quad B_7=\frac{1}{30},\quad B_9=\frac{5}{66},\quad B_{11}=\frac{691}{2730},\quad B_{13}=\frac{7}{6}.$$

Substituting in (1), we find, after some labor, that

$$S_9=\frac{1}{10}n^{10}+\frac{1}{2}n^9+\frac{3}{4}n^8-\frac{7}{10}n^6+\frac{1}{2}n^4-\frac{3}{20}n^2,$$

$$S_{11}=\frac{1}{12}n^{12}+\frac{1}{2}n^{11}+\frac{11}{12}n^{10}-\frac{11}{8}n^8+\frac{11}{6}n^6-\frac{11}{8}n^4+\frac{5}{12}n^2,$$

$$S_{13}=\frac{1}{14}n^{14}+\frac{1}{2}n^{13}+\frac{13}{12}n^{12}-\frac{143}{60}n^{10}+\frac{143}{28}n^8-\frac{143}{20}n^6+\frac{65}{12}n^4-\frac{691}{420}n^2,$$

$$S_{15}=\frac{1}{16}n^{16}+\frac{1}{2}n^{15}+\frac{5}{4}n^{14}-\frac{91}{24}n^{12}+\frac{143}{12}n^{10}-\frac{429}{16}n^8+\frac{455}{12}n^6-\frac{691}{24}n^4+\frac{35}{4}n^2,$$

From the above we have:

$$S_{15}+7S_{13}+7S_{11}+S_9=\frac{1}{16}n^{16}+\frac{1}{2}n^{15}+\frac{7}{4}n^{14}+\frac{7}{2}n^{13}+\frac{35}{8}n^{12}+\frac{7}{2}n^{11}$$
$$+\frac{7}{4}n^{10}+\frac{1}{2}n^9+\frac{1}{16}n^8=\left(\frac{1}{4}n^8+n^7+\frac{3}{2}n^6+n^5+\frac{1}{4}n^4\right)^2$$
$$=\left(\frac{1}{2}n^4+n^3+\frac{1}{2}n^2\right)^4=\left[\frac{n^2(n+1)^2}{2}\right]^4=N^4,\ N,\ \text{an integer}$$

since either n or $n+1$ is even.

Also solved by Charles W. Trigg, Los Angeles.

1312. *Proposed by Joseph Lev, Ithaca, N. Y.*

Evaluate $$\sum_{n=1}^{\infty} ne^{-n},$$

Solved by O. K. DeFoe, St. Louis, Mo.

$$\sum_{n=1}^{\infty} ne^{-n}=\frac{1}{e}+\frac{2}{e^2}+\frac{3}{e^3}+\cdots+\frac{n}{e^n}+\cdots,$$

Now set $$S=\frac{1}{e}+\frac{2}{e^2}+\frac{3}{e^3}+\cdots+\frac{n}{e^n}+\cdots,$$

Therefore $$eS=1+\frac{2}{e}+\frac{3}{e^2}+\cdots+\frac{n}{e^2}+\cdots$$

and

$$eS-S=1+\left(\frac{2}{e}-\frac{1}{e}\right)+\left(\frac{3}{e^2}-\frac{2}{e^2}\right)+\cdots+\left(\frac{n}{e^{n-1}}-\frac{n-1}{e^{n-1}}\right)+\cdots$$

$$(e-1)S=1+\frac{1}{e}+\frac{1}{e^2}+\cdots+\frac{1}{e^{n-1}}+\cdots,$$

This may be summed as a geometric progression

$$(e-1)=\frac{1}{1-\frac{1}{e}}=\frac{e}{e-1}, \text{ From which,}$$

$$S=\frac{e}{(e-1)^2} \qquad \therefore \qquad \sum_{n=1}^{\infty} ne^{-n}=\frac{e}{(e-1)^2}.$$

Also solved by Baris Garfinkel, Buffalo, N. Y., Charles W. Trigg, Los Angeles, Calif., and the proposer.

1313. *Proposed by Charles W. Trigg, Cumnock College, Los Angeles, Calif.*

A polygon whose sides are a, $2a$, $3a$, $4a$, and $5a$ is inscribed in a circle. Find the area of the circle.

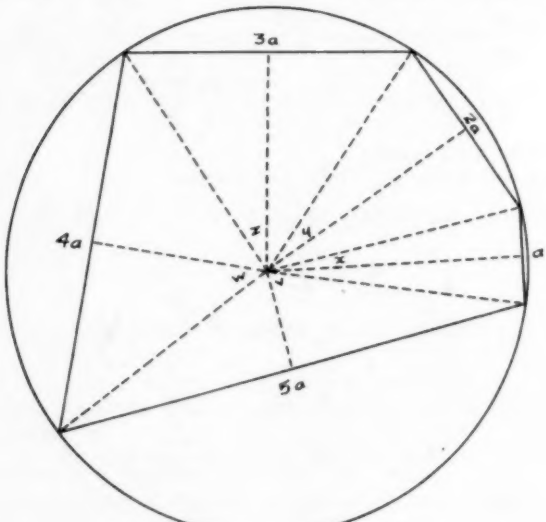
Solved by the Proposer

Let the central angles subtended by the chords a, $2a$, $3a$, $4a$ and $5a$ be $2x$, $2y$, $2z$, $2w$, and $2v$, respectively; and let the radius of the circle be r.

Evidently, $2x=2x$, $2y>2(2x)$, $2z>3(2x)$, $2w>4(2x)$ and $2v>5(2x)$, whence

$$2x+2y+2z+2w+2v=360°>30x, \quad \text{and} \quad x<12°,$$

In a unit circle, with chords in the ratios 1:2:3:4:5, the sines of the half-angles subtended by the chords are equal to the half-chords, which are in the same ratio. If these chords are the sides of an inscribed polygon,



the sum of the half-angles is 180°. Hence, with the aid of a five-place table of natural functions (Macmillan), the magnitude of the chords may be found by a process of successive approximation. Using successive approximations for x of 10°, 11°, 10°40′ and 10°36′, we eventually arrive at,

	∠	sin ∠	∠	sin ∠	∠	sin ∠
x	10°36′10″	0.18400	10°36′ 8″	0.18399	10°36′ 6″	0.18398
y	21°35′33″	0.36800	21°35′29″	0.36798	21°35′24″	0.36796
z	33°30′15″	0.55200	33°30′ 8″	0.55197	33°30′ 0″	0.55194
w	47°23′30″	0.73600	47°23′18″	0.73596	47°23′ 6″	0.73592
v	66°55′33″	0.92000	66°55′ 5″	0.91995	66°54′40″	0.91990
$x+y+z+w+v$	180° 1′ 1″		180° 0′ 8″		179°59′16″	

Hence,

$$r:\tfrac{1}{2}a = 1:0.18399$$

$$r = 2.7175a$$

$$\text{Area of circle} = \pi r^2 = \pi(2.7175a)^2 = 23.200a^2.$$

1314. *Proposed by Reader*

The lines joining the opposite points of contact of a circumscribed quadrilateral are perpendicular to each other.

Solved by W. E. Buker

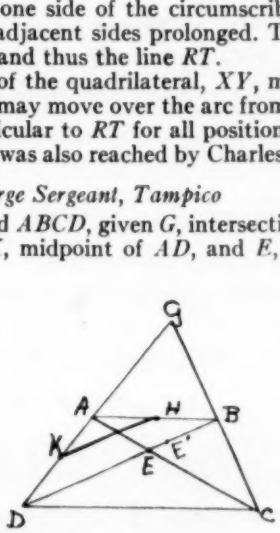
The theorem is obviously false. For, suppose the circle, with center, O, fixed, and that AB is one side of the circumscribed quadrilateral, while BM and AN are two adjacent sides prolonged. The three points of contact, R, S, T are fixed and thus the line RT.

But the fourth side of the quadrilateral, XY, may vary, and the point of tangency, P, of XY may move over the arc from R to T. It is clear that PS cannot be perpendicular to RT for all positions of P.

A similar conclusion was also reached by Charles W. Trigg, Los Angeles.

1315. *Proposed by George Sergeant, Tampico*

Construct a trapezoid $ABCD$, given G, intersection of non parallel sides, H, midpoint of AB, K, midpoint of AD, and E, the intersection of the diagonals.



Solved by H. Hansen Smith, Battle Creek, Iowa

EDITOR'S NOTE: The construction of this problem is only possible when G, H, and E are collinear. In a later issue a proposed problem involving the fact that these points for a trapezoid are collinear will be offered. See Problem 1337.

1. Draw GK and KH. Draw ED parallel to KH.
2. Lay off $KA = KD$, and draw AH cutting DE produced at B.
3. Draw DC parallel to AB and draw GBC.
4. $ABCD$ is the required trapezoid.
5. Draw AC letting it cut BD at E'.

6. GHE is a straight line, (See Note).
7. GHE' is also a straight line.
8. E and E' both lie on GH produced and on DB.
9. Hence E and E' coincide and AC passes through E.

Also solved by Charles W. Trigg, Los Angeles. Several constructions were offered, bit no proofs submitted.

1300. *Proposed by W. E. Buker, Leetsdale, Pa.*

Find the volume of the tetrahedon whose edges are a, b, c, d, e, f.

Solved by Charles W. Trigg, Cumnock College, Los Angles, Calif.

EDITOR'S NOTE: This is the first solution offered to this problem which was proposed in an earlier issue.

The edges of the tetrahedron $VNMP$ are a, b, c, d, e, and f as shown in the figure. With a as radius and M as center describe a sphere cutting b (or b extended) at Y and c (or c extended) at X. Through VM construct a plane perpendicular to NMP and cutting it in RM. Draw VT perpendicular to RM. Then VT is perpendicular to plane NMP and $VT = h$ is the altitude of the tetrahedron on base BMP. Planes VMN, NMP, VMP and VMR intersect the sphere in arcs VY $(=x=\angle VMN)$, YX $(=v=z+w=\angle NMP)$, VX $(=y=\angle VMP)$ and VR $(=p=\angle VMT)$, respectively.

Spherical triangles VYR and VRX are right triangles, hence

$$\cos x=\cos p\cos z \quad \text{and} \quad \cos y=\cos p\cos w.$$

$$\cos v=\cos(z+w)=\cos z\cos w-\sin z\sin w$$

$$=\frac{\cos x\cos y-\sqrt{(\cos^2 p-\cos^2 x)(\cos^2 p-\cos^2 y)}}{\cos^2 p}.$$

$$\cos^2 p=\frac{\cos^2 x+\cos^2 y-2\cos x\cos y\cos v}{1-\cos^2 v}.$$

$$\sin p=\sqrt{\frac{1+2\cos x\cos y\cos v-\cos^2 x-\cos^2 y-\cos^2 v}{1-\cos^2 v}}. \tag{1}$$

Applying the law of cosines to the faces of the tetrahedron,

$$\cos x=\frac{a^2+b^2-d^2}{2ab},\quad \cos y=\frac{a^2+c^2-e^2}{2ac},\quad \text{and}\quad \cos v=\frac{b^2+c^2-f^2}{2bc}. \tag{2}$$

Hence,

$$\sin p=\sqrt{\frac{4a^2b^2c^2+(b^2+c^2-f^2)(a^2+b^2-d^2)(a^2+c^2-e^2)}{a^2\left[4b^2c^2-(b^2+c^2-f^2)^2\right]}} \tag{3}$$

$$\sqrt{\frac{-a^2(b^2+c^2-f^2)^2-c^2(a^2+b^2-d^2)^2-b^2(a^2+c^2-e^2)^2}{a^2\left[4b^2c^2-(b^2+c^2-f^2)^2\right]}}.$$

(4) In $\triangle VTM$, $h=a\sin p$.

(5) By Heron's formula,

$$\text{area } \triangle NMP=\sqrt{s(s-b)(s-c)(s-f)}$$

$$=\tfrac{1}{4}\sqrt{(b+c+f)(b+c-f)(b-c+f)(c+f-b)}$$

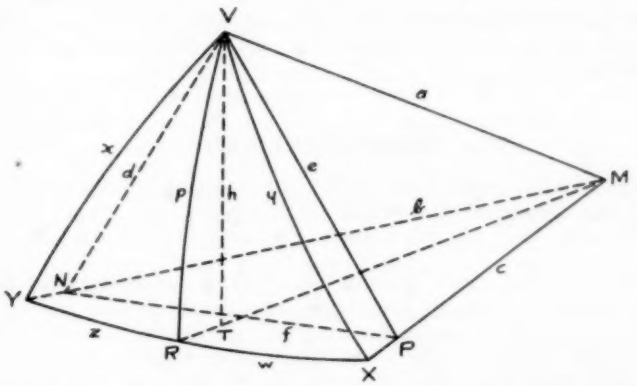
$$=\tfrac{1}{4}\sqrt{4b^2c^2-(b^2+c^2-f^2)^2}$$

(6) V (= volume of tetrahedron) $=\frac{1}{3}h$ (Area $\triangle NMP$).

Substituting (3) in (4) and the result and (5) in (6) and simplifying the resultant equation.

$$V=\frac{1}{12}\sqrt{\begin{aligned}&a^2f^2(b^2+c^2+d^2+e^2-a^2-f^2)+b^2e^2(a^2+c^2+d^2+f^2-e^2-b^2)\\&+c^2d^2(a^2+b^2+e^2+f^2-c^2-d^2)-a^2b^2d^2-a^2c^2e^2\\&-b^2c^2f^2-e^2d^2f^2.\end{aligned}}$$

That is, the volume of a tetrahedron is equal to one-twelfth the square root of the sum of the squares of the products of the opposite edges (each square being multiplied by the sum of the squares of the other four edges



diminished by the sum of the squares of the edges first considered) diminished by the sum of the squares of the products of the edges of each face.

HIGH SCHOOL HONOR ROLL

The editor will be very happy to make special mention of high school classes, clubs, or individual students who offer solutions to problems submitted in this department. Teachers are urged to report to the Editor such solutions.

For this issue the Honor Roll appears below:

1315. H. Hansen Smith, Battle Creek, Iowa.

PROBLEMS FOR SOLUTIONS

1328. *Proposed by Charles Louthan, Columbus, Ohio.*

Find the nature of the path of a point P, 2 in. from the center of a circle M, of diameter 20 in. as M rolls internally on a circle of radius 20 in.

(From Griffith's Mathematical Analysis)

1329. *Proposed by Editor.*

Prove that for $\theta<45°$, $\cot\theta+\cot(45°-\theta)>4$.

1330. *Proposed by Margaret Joseph, Milwaukee, Wis.*

In making war bread a mixture of rye and corn meal was used. From 100 lbs. of rye flour a certain amount was taken and replaced by corn meal. Later, from the mixture the same amount was removed again re-

placed by corn meal. The resulting mixture was 16 pts. rye to 9 pts. corn. What were the proportions of the first mixture?

1331. *Proposed by W. E. Beeker, Leetsdale, Pa.*

Prove or disprove that the altitudes of a tetrahedron are concurrent.

1332. *Proposed by O. K. DeFoe, St. Louis. Mo.*

Consider a clock whose hour, minute and second hands rotate about a common axis. At what time after twelve o'clock will the hands make equal angles with each other, i.e., angles of 120°?

1333. *Proposed by Charles W. Trigg, Cumnock College, Los Angeles, Calif.*

What fundamental relationship exists between the medians (the lines joining the vertices to the centroids of the opposite faces) of a tetrahedron and its edges?

BOOKS RECEIVED

College Physics, by Arthur L. Foley, Professor of Physics and Head of the Physics Department, Indiana University. Cloth. Pages viii+759. 14×21.5 cm. 1933. P. Blakiston's Son and Co. Philadelphia. Price $3.75.

A Combined Laboratory Manual and Workbook in Biology, by Roy E. Davis, Teacher of Biology, East High School, Aurora, Illinois and Ira C. Davis, Assistant Professor in the Teaching of Science, University of Wisconsin. Paper. Pages xi+237. 19×25.5 cm. 1933. Mentzer, Bush and Company, 2210 South Park Way, Chicago, Ill. Wholesale price is 60 cents per copy, list price 80 cents.

Second Year Algebra, by David A. Rothrock, Professor of Mathematics, Indiana University, and Martha Anne Whitacre, Head of Mathematics Department in the Junior and Senior High Schools, Richmond, Indiana. Cloth. Pages xiv+272. 12×19 cm. 1933. Charles Scribner's Sons, 597 Fifth Avenue, New York, N. Y. Price $1.12.

Mechanics and Applied Heat, by S. H. Moorfield, Head of the Mechanical Engineering Department, Wigan and District Mining and Technical College and H. H. Winstanley, Lecturer in Mechanical Engineering, Wigan and District Mining and Technical College. Cloth. Pages viii+328. 12×18.5 cm. 1933. Longmans, Green and Company, 55 Fifth Avenue, New York, N. Y. Price $1.65.

Differential Equations, by Nelson Bush Conkwright, Associate in Mathematics, The University of Iowa. Cloth. Pages xii+234. 12.5×18.5 cm. 1934. The Macmillan Company, 60 Fifth Avenue, New York, N.Y. Price $1.90

The Biology of Bacteria, by Arthur T. Henrici, Professor of Bacteriology, University of Minnesota. Cloth. Pages x+472. 14.5×22.5 cm. 1934. D. C. Heath and Company, 285 Columbus Avenue, Boston, Massachusetts. Price $3.60.

The Alpha Individual Arithmetic. Book Seven, Part II, by The Supervisory Staff of The Summit Country Day School, Cincinnati, Ohio. Illustrated by Cornelia Hoff. Paper. Pages vi+186. 16.5×23 cm. 1934. Ginn and Company, 15 Ashburton Place, Boston, Massachusetts. Price 48 cents.

Tables of the Higher Mathematical Functions, Computed and Compiled under the Direction of Harold T. Davis, Indiana University and the Cowles Commission for Research in Economics with twenty cooperators.

Volume I. Cloth. Pages xiii+377. 17×25 cm. 1933. The Principa Press, Inc., Bloomington, Indiana.

Tables of Integrals and Other Mathematical Data, by Herbert Bristol Dwight, Professor of Electrical Machinery, Massachusetts Institute of Technology. Cloth. Pages viii+222. 14×21.5 cm. 1934. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price $1.50.

Plane Trigonometry and Tables, by William Anthony Granville, Formerly President of Pennsylvania College. Revised by Percey F. Smith, Professor of Mathematics in Yale University and James S. Mikesh, Master in Mathematics in Lawrenceville School. Cloth. Pages xi+212 +47. 14.5×23.5 cm. 1934. Ginn and Company, 15 Ashburton Place, Boston, Massachusetts. Price $1.60.

Exercises and Tests in Intermediate Algebra, by David Eugene Smith, William David Reeve, and Edward Longworth Morss. Paper. 112 pages. 19.5×24.5 cm. 1934. Ginn and Company, 15 Ashburton Place, Boston, Massachusetts. Price 36 cents a copy.

Handbook of Frogs and Toads, by Anna Allen Wright and Albert Hazen Wright. Professor of Zoology, Cornell University. Leather. Pages xi+231. 13.5×20 cm. 1933. Comstock Publishing Company, Inc., 124 Roberts Place, Cornell Heights, Ithaca, New York. Price $2.50.

Physical Optics, Third Edition, by Robert W. Wood, Professor of Experimental Physics in the Johns Hopkins University. Cloth. Pages xvi+846. 13×22 cm. 1934. The Macmillan Company, 60 Fifth Avenue, New York. Price $7.50.

College Physical Science, by Paul McCorkle, Professor of Physics and Physical Science and J. Arthur Lewis, Professor of Chemistry and Physical Science, State Teachers College, West Chester, Penn. Cloth. Pages ix+327. 14×21 cm. 1934. P. Blakiston's Son and Co., 1012 Walnut St., Philadelphia. Price $2.00.

A First Book in Chemistry, by Robert H. Bradbury, Ph.D. Cloth. Pages x+633. 12.5×20 cm. 1934. D. Appleton-Century Company, 35 West 32nd Street, New York. Price $1.80.

BOOK REVIEWS

College Physics, by Arthur L. Foley, Professor of Physics and Head of the Physics Department, Indiana University. Cloth. Pages viii+759. 14×21.5 cm. 1933. P. Blakiston's Son and Co. Philadelphia. Price $3.75.

The author's aim as set forth in the preface is not to impart facts but to arouse and maintain interest and to develop the scientific method of reasoning. In carrying out this plan the author makes effective use of three methods. First, he writes in a straightforward, fluent manner and carefully avoids undue condensation. The importance of rigorous thinking is emphasized and mathematical symbolism is reduced to a minimum. The laws and principles are expressed in words and set in heavy type. The second method is the use of numerous and well chosen drawings. Practically all of the figures were prepared especially for the text and they set a new standard of excellence for American textbooks. Although the total number of figures does not exceed that in other similar books, the reader is more aware of their presence. The figures illustrating the elastic properties of solids, and those in the chapters on sound and on diffraction deserve special mention. They should arouse the interest and admiration of any student who is not hopelessly apathetic to scientific subjects. The third tool that is used to advantage, is the popular one of numerous ques-

tions and problems. The question of whether or not answers should be given for numerical problems is a difficult one for the textbook writer. The author solves this difficulty in a simple, yet original manner. The problems are numbered consecutively, but are arranged in pairs so that each member of a pair involves the same physical principles. Answers are given for one member of each pair.

The choice and arrangement of subject matter is the traditional one with major emphasis placed upon "classical" physics. The general concepts of the nuclear atom and of the electron theory, however, are introduced early and used throughout the book. In the writer's opinion, the oustanding chapters are those on sound and on the wave properties of light. The diffraction photographs of Professor Hufford are exceptionally good. The treatment of sound is clear, thorough and modern. Architectural acoustics receives adequate attention instead of being dismissed with a few remarks. The "bel" and "deci-bel" are defined and their significance explained. All teachers of general physics, including those in secondary schools, will find many valuable suggestions in these chapters.

The program of ample discussion followed in the earlier parts of the book and the limitations of space necessitated a drastic curtailment in the treatment of the recent developments in physics. Only *25 pages* are devoted to the combined topics of radioactivity, x-rays, crystal structure, photoelectricity, the Bohr atom and relativity. Miller's ether drift experiments are described, but the results of Joos and St. John which are not in agreement with those of Miller are not mentioned.

Many teachers, no doubt, feel that new developments in physics are over emphasized in many recent texts. This group will receive the present book with enthusiasm. Those adhering to the opposite opinion may, of course, cover modern developments by means of lectures and assigned reading, since several excellent books on the field are now available.

F. G. TUCKER

The Administration of Mathematics in Secondary Schools, by Ernest R. Breslich, Associate Professor of the Teaching of Mathematics, The University of Chicago. Pages vii+407. 1933. The University of Chicago Press. Price $3.00.

The Administration of Mathematics in Secondary Schools is the third volume of a series dealing with the problems related to the teaching of mathematics in secondary schools. While the first two volumes deal with the technique and the specific problems of teaching, the third deals with specific problems in the administration and supervision of teaching. It is designed primarily to aid the supervisor of instruction, and the department head, but it can be of equal value to the teacher. The following list of chapter headings presents the scope of the book, with the reactions of the reviewer to the discussion.

I. The Supervision of the Department. Treats directly and effectively with the functions of a supervisor in his relations to the department of mathematics in a theoretically large school where such supervision is possible.

II. A Program of Departmental Testing and Research. The chapter classifies the various types of tests and scales, discusses their administration and the analysis of results to improve instruction.

III. Providing for Individual Differences. Presents an adequate treatment of (1) the various plans of teaching, such as the Winnetka Plan, and the Dalton Plan, and (2) the grouping of pupils into more homogeneous classes for the purpose of more effective teaching.

IV. Choosing the Textbook. Presents the recommendations of the various writers in the field for the selection of a suitable textbook. Points out specifically desirable qualities to keep in mind when judging a textbook for adoption.

V. Bases for the Determination of the Aims and Purposes of Teaching Mathematics. This chapter is a complete discussion of the nature, and classification of mathematical aims of instruction, and of the methods and bases used for the determination of such aims.

VI. Methods of Selecting Materials for Teaching Purposes. Here are presented various methods for determining the nature and kind of mathematics used in various fields and activities in order to select instructional materials to be organized into mathematical courses. It is an excellent digest of the reports on the problem.

VII. Organization of the Instructional Materials of Geometry.

VIII. Organization of the Content of Algebra. The outlines suggested in both of these chapters are those used in the author's own textbooks.

IX. The Correlation of Mathematical Subjects. The chapter presents both the advantages and the disadvantages of correlating various mathematical courses, and of coordination between mathematics and physical sciences.

X. Planning the teaching of a Body of Instructional Materials. The chapter illustrates the teaching of a unit in terms of "The Unit on Quadrilaterals." The procedure is that used in the University of Chicago High School, with which the author has been associated for a number of years.

XI. Articulation of Junior and Senior High School Mathematics. Presents specific arguments for the fullest cooperation between the teachers of mathematics in junior and senior high schools.

XII. Unified Mathematics and the Changing Curriculum. This is a resume of the discussions presented in Chapters IX, and XI.

The distinguishing features of the book are:

1. Excellent digest of the reports, articles, analyses and experiments dealing with the various problems in the teaching of mathematics.
2. An extended bibliography at the end of each chapter, dealing with the various phases of the major problem treated in the chapter.
3. Concreteness. Wherever possible the theory has been made practical and illustrated by means of a specific case, treated in detail.
4. Impartially the author presents the advantages and disadvantages of plans presented for discussion without imposing his own beliefs upon the reader.

This book together with its other two companions present an analysis of the problems in the determination of aims, selection of materials, organization, teaching, and evaluation, an undertaking for which the teachers of mathematics are indebted to Professor Breslich. No mathematical library is complete without the three volumes. J. S. GEORGES

A Textbook of Physics, Vol. III, Electricity and Magnetism, by E. Grimsehl. Edited by R. Romaschek, Professor of Physics, the University of Marburg. Authorized Translation from the Seventh German Edition by L. A. Woodward. Cloth. Pages xiv+685. 806 figures. 14.5×22 cm. Blackie and Son, Limited, 50 Old Bailey, London, E.G.4. Price 25s. net.

Here is a textbook of electricity that will satisfy the needs of any college class studying the general field of electricity and magnetism. The theory is carefully developed and fully explained. Many practical applications are woven into the text in such a way that the student is kept aware that he is at work upon theory that is applicable to the explanation of common

phenomena and useful machines and processes. In general the author has followed the historical development of each topic but he quickly carries the student through the early stages to the most recent investigations. Attention is thus directed away from ancient history and toward a developing subject—a very excellent means of stimulating thought and original investigation. The experimental side of the subject is stressed throughout. The author has a very pleasant way of pointing out the observations and contributions of each investigator. This is done in the body of the text when the observations have a direct bearing on the theoretical discussion. Historical footnotes are used to give additional information about the investigator and his contributions.

This book contains a great wealth of material. High school teachers will find it excellent for their personal use and as a reference for students who have questions beyond the realm of secondary school physics. College instructors will find it well adapted for use in classes in electricity in advance of general college physics. It might well replace the highly theoretical texts used in many colleges for the undergraduate course in electricity, since in many cases the text used is too far above the heads of the student. Several pages of problems and exercises are given in the back of the book.

G. W. W.

Introductory Physics, by L. Grant Hector, Professor of Physics in the University of Buffalo. Cloth. Pages xv+373. 13.5×21.3 cm. 1933. American Book Company, 330 East 22nd Street, Chicago, Illinois. Price $3.00.

Here is a textbook of a new type—something out of the ordinary. Brevity is the first difference. However not only has the subject matter been greatly decreased but the arrangement is not in the traditional order. In appearance also there is such marked deviation from the usual form that it would not at first glance be taken as a physics text. One size of type is used throughout except in the legends for figures; italics are used for the introductory paragraphs of each chapter for the literal parts of equations and formulae, and rarely for technical expressions; no bold face type is used. Nearly all the figures are line drawings. The aim of the author has been to give the general student a perspective of the subject and to stimulate him to further study of physics. It is frankly admitted that science majors will have to follow this course with a second year of physics in order to acquire many of the fundamentals.

The text consists largely of theoretical discussions of elementary principles. Practically all historical material has been eliminated even to the exclusion of the names of investigators. No attempt has been made to give any of the details of investigations or the practical operation of physics apparatus. Use of this book alone will give the student very little knowledge and appreciation of the long and difficult path of physical investigation. Its important characteristics are (1) elimination of detail to prevent loading the general student down with a mass of material uninteresting and useless to him; (2) focusing attention on a few fundamentals in order to give thorough understanding of these few rather than a smattering of knowledge about many things; (3) use of modern developments to hold interest and stimulate further study.

It is evident that the author expects the text to be supplemented by use of handbooks, dictionaries, other references and supplementary reading. No doubt also the student is expected to get much from the lecture, class, and laboratory work. Where a good grade of teaching is done with well organized demonstrations and good library facilities this text should help to solve some of the difficulties of students and teachers now completely swamped by the tremendous mass of material in general physics. Where

students must lean heavily on the adopted text this book may prove inadequate. All college teachers of physics recognize the fact that at least in many places something is wrong with the course. This book attempts to right the wrong and merits examination.

G. W. W.

Provision for Individual Differences, Marking, and Promotion, by Roy O. Billett; Monograph No. 13 of the National Survey of Secondary Education, bulletin, 1932, No. 17 of the United States Department of The Interior. 471 pages. For sale by the Superintendent of Documents, Washington, D. C. 40 cents.

This is a study of twenty-eight or more different way of providing for individual differences, the reports being obtained from 8594 schools throughout the entire country. The monograph begins with a review of the past literature and experiments. Teachers may be surprised to hear that homogeneous grouping benefits the slow pupil more than the bright pupil and that the benefits decrease as the pupils' intelligence quotients increase. This is doubtless correct if we merely consider that the dull pupil's chances for failing are decreased, but it does not allow sufficiently for the great number of additional topics that a bright group can cover when its progress is not impeded by dull pupils.

One chapter contains a study of the criterions that are used in classifying the pupils. The smallest schools tend toward the exclusive use of teachers' marks as a single basis, and the largest schools tend toward results from a mental test. Another chapter treats the differentiation of teaching procedures with the groups, the amount of misplacement, and the modification of courses for the groups. Evidently the greatest need is in the modification of the courses.

Besides the tabulation and comparison of a great deal of data, the monograph contains a number of reports from various schools telling exactly what is being done. Here is a generous amount of specific information all of which is intensely interesting. The reports from the schools would have been still more valuable if they had stated the nature of the community around the school. A school in a prosperous suburban district, for example, is quite different from one surrounded by steel mills. There is also a chapter dealing with the organization of special classes, such as remedial classes and opportunity classes.

To present a picture of the twenty-eight or more methods of providing for individual differences the monograph devotes two hundred pages to the Morrison plan, the Dalton plan, the Winnetka plan, the plan by unit assignments and a miscellaneous collection of other plans. It states clearly the characteristic features of each together with statistics showing how extensively each plan is used. Most government bulletins are dull reading; this one is an exception. For forty cents the teacher can get a textbook on one of the most important phases of modern education.

JOSEPH A. NYBERG

Amateur Telescope Making. Edited by Albert G. Ingalls, Associate Editor of Scientific American, with a foreword by Dr. Howard Shapely, Director of Harvard College Observatory. Cloth. Pages xii+500. 13.5×19 cm. 1933. Scientific American Publishing Co. Price $3.00.

For the amateur scientific hobby hunter the making of a telescope has in the past been generally regarded as within the capacity of the professional optician only. While it is true that prior to the publication Amateur Telescope Making there was only one book in the English language available to the amateur, it was more of a "bogey" than lack of information

which prevented us from studying the heavens through the work of our own hands. The writers of Amateur Telescope Making, writing from their own experience, have looked this bogey straight in the face and have found that with the average amount of mechanical skill a really scientific instrument could be made.

The book comprises a series of articles which appeared in the Scientific American for the past few years. The author of each section writes from experience, making the entire work very readable. Constructional detail and procedure are given rather explicitly, however not too step-like to discourage initiative on the part of the amateur.

The greater portion of the book is devoted to the construction of the reflecting telescope which is the amateur's telescope in distinction to the refracting telescope which presents more and greater bogeys. However, those with some knowledge of lens corrections will find sufficient detail for the construction of the latter type.

For the more truly pretentious amateur who wishes to do solar research work the editor has included a section on Solar Phenomena and the construction and use of the spectroheliograph and spectrohelioscope.

A justifiable pride comes from fashioning glasses with your own hands besides the opportunity to indulge your curiosity in their use.

CLARENCE RADIUS

Plane Trigonometry, by William L. Hart, Ph.D., Professor of Mathematics, University of Minnesota. Cloth. Pages v+186. 1933. D. C. Heath and Company, New York.

This book offers a concise treatment of plane trigonometry giving recognition to both the numerical and analytical sides of the subject. The book is flexible in nature and many topics may be omitted at the discretion of the instructor without destroying the continuity of the subject. The chapters on the solutions of triangles contain many interesting applications.

The trigonometry of the acute angle and its applications are presented before the general angle, and identities and equations are particularly emphasized. Both four-place and five-place tables are given and the natural values of secants and cosecants are included.

Review exercises appear frequently and a summary of 200 problems is given at the end of the text. A large sized page is used since the author feels that this permits incorporation of several useful devices in the tables and insures maximum legibility in the most important pages. The reviewer is of the opinion that this advantage is offset by the disadvantage of using an unwieldy book. Even though the pages are large there seems to be considerable crowding. The book should prove useful to teachers of trigonometry.

C. A. STONE

New High School Arithmetic, Revised, by Webster Wells and Walter W. Hart, Associate Professor of Mathematics, School of Education and Teacher of Mathematics, Wisconsin High School, University of Wisconsin. Cloth. Pages xiii+357. 1933. D. C. Heath and Company, New York.

This book is designed for all types of pupils in the high school regardless of the course they intend to follow. It is well fitted for use in an arithmetic course of high school level, and one feels that the appearance of such a book may stimulate the addition of a much needed arithmetic course in the secondary school.

Not only does the text contain ample drill material for the purpose of developing skill in computation but it contains an adequate amount of

material for those pupils planning to enter the business world. The book is attractive, the material is well organized, and it should find favor with teachers of business arithmetic and those advocating arithmetic in the high school.

C. A. STONE

Study Guides and Unit Tests (To accompany *Living Geography*), by M. E. Branom, Head of Department of Geography, Harris Teachers College, St. Louis, Missouri. Macmillan Company, New York, 1933.

"Study Guides—a Teaching Aid."

The teacher who considers study guides and workbooks as helpful tools in teaching Geography will find in this set of books a well-organized and complete list of questions and suggestions. The teacher and the student will find a mass of additional material in *Living Geography* by using these guides.

These are four paper-bound pamphlets, 11″×8½″, of ninety-six pages each. These represent two books of two parts each. The arrangement of material follows exactly the units of "Living Geography." Book I on "How Countries Differ" has nine units thoroughly covered in pamphlets one and two. Book II "Why Countries Differ" is covered completely in thirteen units by pamphlets three and four. At the end of each unit of study are questions for the review of the unit.

The experienced and wide-awake teacher will find valuable suggestions in these guides which will help to make the already successful course more interesting and understandable. The inexperienced and average teacher will find in these guides a wealth of suggestions and indispensable aids.

L. F. FOUNTAIN

A Combined Laboratory Manual and Workbook in Biology, by Roy E. Davis, East High School, Aurora, Illinois, and Ira C. Davis, Asst. Professor in the Teaching of Science, University of Wisconsin. 237 pages, 61 figures. 1933. Mentzer, Bush and Company, Chicago and New York.

This new guide to the study of biology in high school classes is built on an organization all its own and does not depend upon that of any one text. Seven different texts are used as reference regularly throughout the course. This does not mean that all must be used, but if any one of the seven is used as a text the others, or as many as may be available, may be used for additional reading. The course can be followed with any good high school biology as a text.

The first of the four units of the course deals with fundamental aspects of the physical and chemical basis of plant and animal structure and function. The second unit is devoted to a study of animal life. An eminently commendable feature of the course here as elsewhere is the emphasis placed on head work rather than on hand work. Structure is not studied for its own sake, but rather as to its bearing on function, classification, and economic phases of the subject. Directions and questions similar to the following are characteristic: "Give two examples of internal poison insecticides." "This class is used for insects with . . . mouth parts." "What two important things must be known about insects, to be able to understand the methods of control that are needed?" Unit III is given to a development of understandings of the structure, function, and care of the human body. The same general plan as that used in the study of animal life in the second unit is used in the study of plants in the fourth unit. Many sketches of plant structure are given as a guide to study, but comparatively few are required to be made by the students throughout the course.

The work is made up of ninety-five exercises. In connection with each exercise there is given a list of words for study and specific questions to be answered by pupils. Ample provision is made for class discussion and testing of results. With the tendency toward increased teacher load, prepared material of the type offered in this workbook serves a most useful purpose. The experienced teacher of biology as well as those beginning the work will find in this manual a most time saving and helpful guide to efficient teaching.

JEROME ISENBARGER

Introduction to Biology, by Elbert C. Cole, Professor of Biology, Williams College; Formerly Instructor in Biology, Hartford Public High School, Hartford, Connecticut. 532 pages, 291 figures. 1933. John Wiley and Sons, Inc.

This is an introductory text which places proper emphasis upon the principles of biology rather than undue emphasis on the materials of biology. The development of the subject is taken up under seven divisions as follows: Organisms and Their Environment, Life Activities of Plants, Some Fundamental Processes in Organisms, Man as an Organism, Life Activities of Animals, Control and Improvement of Organisms, and The Kinship of Organisms. Each division uses from two to nine chapters of the text in presenting the subject matter. Among significant headings of chapters which illustrate the emphasis on principles are, "The Interrelations of Organisms," "How Animals Secure Oxygen," "How the Body Prepares Food for Its Use," and "The Business of Keeping Well."

The use of illustrations provides a notable feature of the book in that they are made a part of the context. Detailed legends are given in such a way that the material of the picture or diagram at once becomes a basis for class discussion and a valuable aid in developing definite understandings. Reproductions from photographs are not just pretty pictures, but they are introduced as visual aids to the printed discussion. The excellence of this feature cannot be emphasized too strongly.

Suggested activities in the form of home and laboratory experiments and exercises are given at the end of each chapter under the heading, "Projects." At the end of each chapter, also, there is given a list of related readings to be found in different parts of the book, which have a bearing on the material of the chapter. These cross references can be made a valuable aid, if properly used. Sufficient material is given in the projects so that an incentive is offered the more proficient students to do extra work.

The book recommends itself as a most useful reference in any elementary biology class and should prove an excellent text in the type of course which it develops.

JEROME ISENBARGER

The Elements of Euclid, Edited by Isaac Todhunter with an Introduction by Sir Thomas Heath. Pages xviii+298. 11.5×17.5 cm. Cloth. 1933. J. M. Dent and Sons, Ltd. London and Toronto. E. P. Dutton and Co. Inc., New York.

This edition is No. 891 of Everyman's Library. It is a copy of Todhunter's edition based on Simpson's edition. Todhunter omitted Propositions 27–29 from Book VI. These have been supplied in an appendix. Todhunter's notes on Euclid's Elements are included. Every teacher of geometry should be familiar with this or some other edition of Euclid.

J. M. KINNEY

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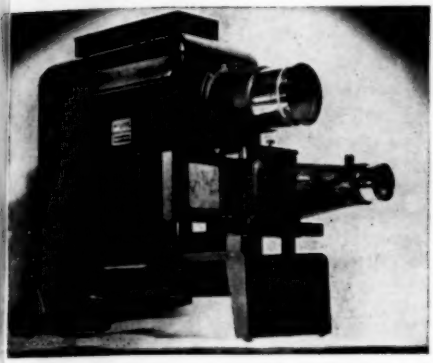

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Brief Course in Plane and Spherical Trigonometry, by H. A. Davis, Ph.D., Assistant Professor of Mathematics, West Virginia University and L. H. Chambers, A.M., Instructor in Mathematics, West Virginia University. Pages 136+96. 15×22.2 cm. 1933. American Book Company, New York. Price with tables $2.00. Without tables $1.50.

This book offers a brief course in Plane and Spherical Trigonometry. Formulas which the student is not likely to use are omitted. The trigonometric functions are first defined for the acute angles. The chapter on the solution of the right triangle follows the chapter on logarithms. There is a rather complete treatment of De Moivre's Theorem, trigonometric series, and hyperbolic functions. In an appendix some facts of the solar system are given. One feature which is especially commendable is the form in which solutions of computation problems are set up. Each number, as a rule, is written but once.

J. M. KINNEY

The New Day Junior Mathematics, Book III, by Vevia Blair, Head of Department of Mathematics, Horace Mann School, Teachers College, Columbia University. Pages xii+430. 1933. Charles E. Merrill Co. New York, N. Y.

This volume, the third of a series of Junior High School Mathematics, contains the essentials of a traditional course in algebra, yet the breadth of treatment and variety of materials make the text adaptable to a progressive course in ninth year mathematics. The first two chapters show by the use of tables, formulas, and graphs how numbers depend on other numbers. This organization and material offer an excellent illustration of a way in which the function concept may be introduced in an elementary manner. Opportunity to develop the power to think functionally is presented frequently throughout the text.

The book offers several other commendable features. The approach to each process is through interest in its eventual use, created by the presentation of a concrete problem situation which challenges solution. The problem material is new, practical, and interesting, with many applications chosen from modern science and industry. Provision is made for individual differences by grading the exercises according to difficulty and separating them into three levels by inconspicuous lines. The book also contains a chapter on Trigonometry, one on the introduction to Geometry, and one on Statistical Measurements.

G. E. HAWKINS

Eleventh Year Mathematics, Intermediate Algebra and Trigonometry, by Joseph B. Orleans, Head of Mathematics Department, George Washington High School, New York City and Hallie S. Poole, Head of Mathematics Department, LaFayette High School, Buffalo, New York. 261 pages, including tables. 1932. Published by D. C. Heath and Co. Price $1.32.

After a thorough examination of this text which supposedly correlates the subject matter of trigonometry and advanced algebra the reviewer was left with the impression that this text is a heterogenous mass of mathematics. Only in the hands of a very skillful teacher would a student emerge from this combined course with a definite organization of trigonometry and advanced algebra. Although the authors quote Percy T. Nunn in the preface one cannot help but feel that the text is the antithesis of anything Nunn, Klein or Moore, and a host of others who have advocated the correlation of mathematics, have wished for in the way of

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mathematics texts. To substantiate these statements, it can be stated again that the text lacks a unifying principle, and this is due to the fact that the authors have reverted to the old fashioned lesson plan. The body of the text contains 99 lessons in mathematics—groups of lessons in trigonometry intersperced between lessons in algebra. The book fails to present a unified whole for this reason and furthermore for the reason that the major topics in the lesson plan are subdivided into smaller parts and nowhere do the authors help the student organize and summarize the materials.

Although ten lessons are devoted to logarithms the authors did not deem it worthwhile to teach the use of the cologarithm which most mathematics teachers agree upon as a very valuable short cut in logarithmic computation. However in all fairness it must be stated that it is given as an additional lesson at the very end of the text.

The subject of trigonometry furnishes an outstanding example of correlation of algebra and geometry and to attempt to correlate more algebra with it as the authors have done is futile. GEO. L. ROYCE

An Elementary Treatise on Differential Equations, by Abraham Cohen, Ph.D., Collegiate Professor of Mathematics, The John Hopkins University. Revised. Cloth, Pages vii + 337. 14.5 × 20.5 cm. D. C. Heath and Company, 285 Columbus Ave., Boston, Massachusetts. Price $2.40. 1933.

This text-book on differential equations is a completely revised edition. The first edition appeared in 1906. The revised edition has been enlarged by the addition of new topics and by an increase in the number of problems. We find many exercises dealing with interesting applications of differential equations to the physical sciences. The typography of this edition is much better than that of the first. J. M. KINNEY

FOR THE TEACHER

The following announcement appeared in a recent tissue of *Current Science:*

A NEW LIST OF FREE MATERIALS

One of the requests most frequently received by the Editor is for lists of free and low cost materials that may be secured from manufacturers, sales organizations, and the like, to be used in class to enrich the textbook presentations. Lists of this nature involve much correspondence, however, and only now and then is a new one published.

During the past summer a colleague of the Editor, Prof. Ullin W. Leavell, Professor of Elementary Education at George Peabody College for Teachers, with the assistance of members of his class, carried out the labors necessary to compile a new list of this type of material, designed particularly to interest students in the Elementary and Grammar Grades.

An examination of the large amount of free material which he received shows that high school science teachers could use most of it very helpfully in their classes, going more deeply, as a rule, into the explanations of the objects, illustrated folders, and the like.

This list prepared by Dr. Leavell is *current up to this fall*, hence most of the firms offering materials at that time still have them available. Several hundred firms and organizations are represented. The list is alphabetically arranged by classes of materials.

One of these lists will be mailed to any teacher who will send a 2c stamp to pay for addressing and postage. Mail the request to *Editor of* CURRENT SCIENCE, *George Peabody College for Teachers, Nashville, Tenn.*

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A FORMULA FOR WIRE RESISTANCE

HOLLIS D. HATCH
English High School, Boston, Massachusetts

The usual way to calculate the resistance of a copper wire is from its specific resistance, length, and circular mils. (cf. Black and Davis p. 333 of Sears p. 435). Another method, original with the author, is of less teaching value but mathematically interesting and depends upon the constant area ratio of the wires in the B. and S. table.

Between #0000 and #40 (the largest and smallest) are 43 intervals corresponding to a cross section ratio of 211,600 to 9.89. The 43rd root of this ratio is 1.261. This means that each wire has a cross section area 1.261 times less than the one above it in the table and an electrical resistance that much more. Now tables (cf. Dull: p. 509) say that 1262 ft. of #9 wire have a resistance of one ohm so let us start from that. The same length of #10 wire would have 1.261 times as much resistance or 1.261 ohms; the same length of #11 would be $(1.261)^2$ times one ohms; or in general we can say that the resistance of 1262 ft. will be $(1.261)^{N-9}$ ohms.

Of course we do not always have 1262 ft. of wire. Five hundred feet, for instance, would have 500/1262 as much resistance. Or in general the resistance is $L/1262$ times that of 1262 ft.

Now let us combine the two ways the resistance varies and we have:

$$R = \frac{L(1.261)^{N-9}}{1262}$$

You notice 1262 is nearly 1000(1.261) so make this substitution and simplify:

$$R = \frac{L(1.261)^{N-9}}{1000(1.261)} = \frac{L(1.261)^{N}}{1000(1.261)^{10}} = \frac{L(1.261)^{N}}{10{,}160}$$

where: L = feet, R = ohms, $N = B$ and S gauge.

Now to get resistance all you need is a slide rule or a table of logarithms. Example: Find the resistance of 4650 ft. #8 Cu wire.

$$\begin{aligned} \log.\ 1.261 &= 0.1007 \\ \log.\ 1.261^{8} &= 0.8056 \\ \log.\ 4650 &= 3.6675 \\ \hline & \quad 4.4731 \\ \log.\ 10160 &= 4.0069 \\ \hline & \quad 0.4662 = \log \text{ of } 2.93 \text{ (ohms)} \end{aligned}$$

This method is more accurate than a slide rule or for that matter than the accuracy with which wires are made commercially.

SPECIAL SPECTACLES ALLOW DIVERS TO SEE CLEARLY UNDER WATER

Special underwater spectacles that allow lifeguards and divers to see clearly beneath the water's surface have been devised by Robert E. Cornish of the University of California's Institute of Experimental Biology.

The normal human eye, developed for vision in air, is a very poor instrument under water as every swimmer knows. The reason is that the contact of water with the cornea robs the eye of about two thirds of its refracting power. This trouble is avoided in divers' outfits by keeping

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